

# Hempel's logic of confirmation

Franz Huber

Received: 28 July 2006 / Accepted: 18 April 2007 / Published online: 2 June 2007  
© Springer Science+Business Media B.V. 2007

**Abstract** This paper presents a new analysis of C.G. Hempel's conditions of adequacy for any relation of confirmation [Hempel C. G. (1945). *Aspects of scientific explanation and other essays in the philosophy of science*. New York: The Free Press, pp. 3–51.], differing from the one Carnap gave in §87 of his [1962. *Logical foundations of probability* (2nd ed.). Chicago: University of Chicago Press.]. Hempel, it is argued, felt the need for two concepts of confirmation: one aiming at true hypotheses and another aiming at informative hypotheses. However, he also realized that these two concepts are conflicting, and he gave up the concept of confirmation aiming at informative hypotheses. I then show that one can have Hempel's cake and eat it too. There is a logic that takes into account both of these two conflicting aspects. According to this logic, a sentence  $H$  is an acceptable hypothesis for evidence  $E$  if and only if  $H$  is both sufficiently plausible given  $E$  and sufficiently informative about  $E$ . Finally, the logic sheds new light on Carnap's analysis.

## 1 Hempel's conditions of adequacy

In his "Studies in the Logic of Confirmation" (1945) Carl G. Hempel presented the following conditions of adequacy for any relation of confirmation (names for 3.1 and 3.2 added). For any sentence (observation report)  $E$  and any sentences (hypotheses)  $H, H'$ :

1. Entailment Condition  
If  $E$  logically implies  $H$ , then  $E$  confirms  $H$ .

---

The author is grateful to Peter Brössel for comments on an earlier version of this paper.

---

F. Huber (✉)  
Division of Humanities and Social Sciences, California Institute of Technology, 1200 East California  
Blvd, Pasadena, CA, USA  
e-mail: franz@caltech.edu

## 2. Consequence Condition

If the set of all hypotheses confirmed by  $E$ ,  $\mathcal{H}_E$ , logically implies  $H$ , then  $E$  confirms  $H$ .

### 2.1 Special Consequence Condition

If  $E$  confirms  $H$  and  $H$  logically implies  $H'$ , then  $E$  confirms  $H'$ .

### 2.2 Equivalence Condition

If  $E$  confirms  $H$  and  $H'$  is logically equivalent to  $H$ , then  $E$  confirms  $H'$ .

## 3. Consistency Condition

$E$  is compatible with the set  $\mathcal{H}_E$  of all hypotheses it confirms.

### 3.1 Special Consistency Condition

If  $E$  is consistent and confirms  $H$ , and if  $H$  logically implies  $\neg H'$ , then  $E$  does not confirm  $H'$ .

### 3.2 Consistent Selectivity

If  $E$  is consistent and confirms  $H$ , then  $E$  does not confirm  $\neg H$ .

## 4. Converse Consequence Condition

If  $E$  confirms  $H$  and  $H'$  logically implies  $H$ , then  $E$  confirms  $H'$ .

Condition 2 entails condition 2.1, which in turn entails condition 2.2; similarly in case of 3. Hempel then showed (Hempel 1945: 104) that the conjunction of 1, 2, and 4 entails his triviality result that every sentence (observation report)  $E$  confirms every sentence (hypothesis)  $H$ . This is clear since the conjunction of 1 and 4 already implies this. By the Entailment Condition,  $E$  confirms  $E \vee H$ . As  $H$  logically implies  $E \vee H$ , the Converse Consequence Condition yields that  $E$  confirms  $H$ , for any sentences  $E$  and  $H$ .

Since Hempel's negative result, there has hardly been any progress in developing a logic of confirmation.<sup>1</sup> One reason for this seems to be that up to now the predominant view on Hempel's conditions is the analysis Carnap gave in his *Logical Foundations of Probability* (1962), §87.

## 2 Carnap's analysis of Hempel's conditions

In analyzing the Consequence Condition, Carnap argues that

Hempel has in mind as explicandum the following relation: 'the degree of confirmation of  $H$  by  $E$  is greater than  $r$ ', where  $r$  is a fixed value, perhaps 0 or 1/2. (Carnap 1962: 475; notation adapted)

In discussing the Consistency Condition, Carnap mentions that

Hempel himself shows that a set of physical measurements may confirm several quantitative hypotheses which are incompatible with each other (p. 106). This seems to me a clear refutation of [3.1]. ... What may be the reasons that have led Hempel to the consistency conditions [3.1] and [3]? He regards it as a great advantage of any explicatum satisfying [3] "that it sets a limit, so to speak, to the strength of the hypotheses which can be confirmed by given evidence" ... This argument does not

<sup>1</sup> The exceptions I know of are Flach (2000), Milne (2000), and Zwirn and Zwirn (1996). Roughly, Zwirn and Zwirn (1996) argue that there is no unified logic of confirmation (taking into account all of the partly conflicting aspects of confirmation). Flach (2000) argues that there are two logics of "induction", as he calls it, viz. confirmatory and explicatory induction (corresponding to Hempel's conditions 1–3 and 4, respectively). Milne (2000) argues that there is a logic of confirmation—namely the logic of positive probabilistic relevance – but that it does not deserve to be called a logic.

seem to have any plausibility for *our* explicandum, (Carnap 1962; 476–477; emphasis in original).

which is the classificatory or qualitative concept of (initially) confirming evidence, as Carnap says in §86 of his (1962), that he explicates in terms of positive probabilistic relevance.

But it is plausible for the second explicandum mentioned earlier: the degree of confirmation exceeding a fixed value  $r$ . Therefore we may perhaps assume that Hempel's acceptance of the consistency condition is due again to an inadvertent shift to the second explicandum. (Carnap 1962: 477–478).

Carnap's analysis can be summarized as follows. In presenting his first three conditions of adequacy Hempel was mixing up two distinct concepts of confirmation, two distinct explicanda in Carnap's terminology. These are

1. the concept of (initially) confirming evidence: Carnap explicates this concept by incremental confirmation or positive probabilistic relevance according to which  $E$  confirms  $H$  if and only if  $E$  (has non-zero probability and) increases the probability of  $H$ , i.e.,  $\Pr(H|E) > \Pr(H)$ , and
2. the concept of the degree of confirmation exceeding a fixed value  $r$ : Carnap explicates this concept by absolute confirmation according to which  $E$  confirms  $H$  if and only if the probability of  $H$  given  $E$ ,  $\Pr(H|E)$ , is greater than some value  $r$ .

The special versions of Hempel's second and third condition, 2.1 and 3.1, respectively, hold true for the second explicatum (for  $r \geq .5$ ). However, they do not hold true for the first explicatum. On the other hand, Hempel's first condition holds true for the first explicatum, but it does so only in a *qualified* form (Carnap 1962: 473)—namely only if  $E$  is not assigned probability 0, and  $H$  is not already assigned probability 1.

This means that, according to Carnap's analysis, Hempel first had in mind the explicandum of (initially) confirming evidence for the Entailment Condition. Then he had in mind the explicandum of the degree of confirmation exceeding a fixed value  $r$  for the Special Consequence and the Special Consistency Conditions 2.1 and 3.1, respectively. Finally, when Hempel presented the Converse Consequence Condition, he got completely confused and had in mind still another explicandum or concept of confirmation.<sup>2</sup> Apart from not being very charitable, Carnap's reading of Hempel also leaves open the question what the third explicandum might have been.

### 3 Conflicting concepts of confirmation

The following two notions are central to the plausibility-informativeness theory (Huber 2007a). A relation of confirmation is an *informativeness* relation if and only if the following holds:

If  $E$  confirms  $H$  and  $H'$  logically implies  $H$ , then  $E$  confirms  $H'$ .

A relation of confirmation is a *plausibility* relation if and only if the following holds:

If  $E$  confirms  $H$  and  $H$  logically implies  $H'$ , then  $E$  confirms  $H'$ .

<sup>2</sup> Neither the first nor the second explicatum satisfies the Converse Consequence Condition.

The idea is that a sentence or proposition is the more informative, the more possibilities it excludes. Hence, the logically stronger a sentence, the more informative it is. On the other hand, a sentence is more plausible the fewer possibilities it excludes, i.e., the more possibilities it includes. Hence, the logically weaker a sentence, the more plausible it is. The qualitative counterparts of these two comparative principles are the defining clauses above. If  $H$  is informative relative to  $E$ , then so is any logically stronger sentence  $H'$ . Similarly, if  $H$  is plausible relative to  $E$ , then so is any logically weaker sentence  $H'$ .

The two main approaches to confirmation that have been put forth in the last century are qualitative *Hypothetico-Deductivism* HD and quantitative probabilistic *Inductive Logic* IL. According to HD,  $E$  HD-confirms  $H$  if and only if  $H$  logically implies  $E$  (in some suitable way that depends on the version of HD under consideration). According to IL, the degree of *absolute* confirmation of  $H$  by  $E$  equals the (logical) probability of  $H$  given  $E$ , i.e.,  $\Pr(H|E)$ . The natural *qualitative* counterpart of this quantitative notion is that  $E$  (*absolutely*) IL-confirms  $H$  if and only if the probability of  $H$  given  $E$  is greater than some value  $r$  in  $].5,1)$ , i.e.,  $\Pr(H|E)$  (this is Carnap's second explicatum).

As noted above, this is not the way Carnap defines qualitative IL-confirmation in chapter VII of his (1962). There he requires that  $E$  raise the probability of  $H$ ,  $\Pr(H|E) > \Pr(H)$ , in order for  $E$  to qualitatively IL-confirm  $H$ . Nevertheless, the above is the natural qualitative counterpart of the degree of absolute confirmation. The reason is that later on, the difference between  $\Pr(H|E)$  and  $\Pr(H)$ —however it is measured (Fitelson 1999)—was taken as the degree of *incremental confirmation*. Carnap's proposal is the natural qualitative counterpart of this notion of incremental confirmation. In order to distinguish these two notions, let us say that  $E$  *incrementally confirms*  $H$  if and only if  $\Pr(H|E) > \Pr(H)$ .

HD and IL are based on two *conflicting* concepts of confirmation. HD-confirmation *increases*, whereas (absolute) IL-confirmation *decreases* with the logical strength of the hypothesis to be assessed. More precisely, if  $E$  HD-confirms  $H$  and  $H'$  logically implies  $H$ , then  $E$  HD-confirms  $H'$ . So, as a matter of fact, HD-confirmation aims at *logically strong* hypotheses—HD-confirmation is an informativeness relation. On the other hand, if  $E$  absolutely IL-confirms  $H$  (to some degree  $r$ ) and  $H$  logically implies  $H'$ , then  $E$  absolutely IL-confirms  $H'$  (to at least the same degree  $s \geq r$ ). Hence, as a matter of fact, absolute IL-confirmation aims at *logically weak* hypotheses—absolute IL-confirmation is a plausibility relation.

The epistemic virtues behind these two concepts are *informativeness* on the one hand and *truth* or *plausibility* on the other hand. We want to know what is going on “out there”, and hence we aim at true hypotheses—more precisely, at hypotheses that are true in the world we are in. We also want to know as much as possible about what is going on out there, and so we aim at informative hypotheses—more precisely, at hypotheses that inform us about the world we are in. But usually we do not know which world we are in. All we have are some data. So we base our evaluation of the hypothesis we are concerned with on the plausibility that the hypothesis is true in the actual world given that the actual world makes the data true and on how much the hypothesis informs us about the actual world given that the actual world makes the data true.

If one of two hypotheses logically implies the other, the logically stronger hypothesis excludes all the possibilities excluded by the logically weaker one. The logically stronger hypothesis is thus at least as informative as the logically weaker one. On the other hand, the logically weaker hypothesis is at least as plausible as the logically stronger one. The reason is that all possibilities making the logically stronger hypothesis true also make the logically weaker one true. This is the sense in which the two concepts underlying HD and IL, respectively, are *conflicting*.

#### 4 Hempel vindicated

Turning back to Hempel's conditions, note first that Carnap's second explicatum satisfies the Entailment Condition *without* the second qualification: if  $E$  logically implies  $H$ , then  $\Pr(H|E) = 1$ , for any value  $r$  in  $[0, 1)$ . This holds provided  $E$  does not have probability 0 (this proviso can be dropped by using appropriate alternatives to classical probability measures).

So the following more charitable reading of Hempel seems plausible. When presenting his first three conditions, Hempel had in mind Carnap's second explicandum, the concept of the degree of confirmation exceeding a fixed value  $r$ , or more generally, a plausibility relation. In fact, what Hempel actually seemed to have in mind is the explicandum underlying the satisfaction criterion of confirmation (Hempel 1945: 107–112, esp. 109, and Hempel 1943), which is a plausibility relation satisfying these three conditions. But then, when discussing the Converse Consequence Condition, Hempel also felt the need for a second concept of confirmation aiming at informative hypotheses—explicated, it seems, in terms of the prediction-criterion of confirmation (Hempel 1945: 97–102, esp. 98) and its “quantitative counterpart”, the concept of systemic power (Hempel and Oppenheim 1948: 164–173; 168, fn. 35, for the quote).

Given that it was the Converse Consequence Condition that Hempel gave up in his “Studies in the Logic of Confirmation”, the present analysis makes perfect sense of his reasoning. Though he felt the need for two concepts of confirmation, Hempel also realized that these two concepts are *conflicting*—this is the content of his triviality result. Consequently he abandoned informativeness in favor of plausibility.

Let us check this by going through Hempel's conditions. Hempel's Entailment Condition is that  $E$  confirms  $H$  if  $E$  logically implies  $H$ . In this case, the plausibility of  $H$  given  $E$  is maximal, because the data guarantee the truth of the hypothesis. On the other hand, if  $E$  entails  $H$ ,  $H$  need not inform us about  $E$ .

The Consequence Condition says that if  $E$  confirms a set of hypotheses  $\mathcal{H}$ , then  $E$  confirms every consequence  $H$  of  $\mathcal{H}$ . This condition clearly does not hold of the informativeness concept. It holds without qualification for the plausibility concept only in its restricted form 2.1, the Special Consequence Condition (this is also noted by Carnap 1962: 474–476). The latter condition expresses that plausibility decreases with the logical strength of the hypothesis to be assessed. In particular, if the probability of  $H$  given  $E$  is greater than some value  $r \geq .5$ , then so is the probability of any logical consequence  $H'$  of  $H$ .

The Consistency Condition says that every consistent  $E$  is compatible with the set  $\mathcal{H}_E$  of all hypotheses  $E$  confirms. As before, this condition does not hold true of the informativeness concept. It holds true for the plausibility concept only with a proviso and only in its restricted form 3.1, the Special Consistency Condition: if  $E$  is consistent and confirms  $H$ , then  $E$  confirms no  $H'$  which is not compatible with  $H$ . In this case,  $H'$  logically implies  $\neg H$ , whence the plausibility of  $H'$  is not greater than that of  $\neg H$  (plausibility decreases with logical strength). Given the proviso that no consistent  $E$  confirms both a hypothesis and its negation, the result follows. As noted by Carnap (1962: 478), this proviso is satisfied if plausibility is measured by a probability measure and  $r \geq .5$ : if the probability of hypothesis  $H$  given evidence  $E$  is high ( $> 1/2$ ), then the probability of any hypothesis  $H'$  given  $E$ , where  $H'$  is not compatible with  $H$ , must be low ( $< 1/2$ ). The reason is that the sum of these two probabilities cannot exceed 1.

The culprit, according to Hempel (1945: 103–107, esp. 104–105), is the Converse Consequence Condition: if  $E$  confirms  $H$  and  $H$  is logically implied by  $H'$ , then  $E$  confirms

$H'$ . Clearly, this condition holds for informativeness, but not for plausibility. In fact, it coincides with the defining clause of informativeness relations by expressing the requirement that informativeness increases with the logical strength of the hypothesis to be assessed. The Converse Consequence Condition is satisfied by HD-confirmation and, more importantly, by Hempel's prediction-criterion of confirmation, which is a qualitative version of his measure of systematic power.

## 5 The logic of hypothesis assessment

However, in a sense, one can have Hempel's cake and eat it too. There is a logic of confirmation—or rather, hypothesis assessment—that takes into account both of these two conflicting concepts. Roughly speaking, HD says that a good hypothesis is informative, whereas IL says that a good hypothesis is plausible or true. The driving force behind Hempel's conditions is the insight that *a good hypothesis is both true and informative*. Hence, in assessing a given hypothesis by the available data, one should account for these two conflicting aspects.

According to the logic of hypothesis assessment, a sentence or proposition  $H$  is *an acceptable hypothesis* for evidence  $E$  if and only if  $H$  is at least as plausible as and more informative than its negation relative to  $E$ , or  $H$  is more plausible than and at least as informative as its negation relative to  $E$ .

In order to formalize this notion of acceptability we have to employ some formal tool. Given that Carnap formulated his account in terms of probabilities, it is convenient (for reasons of comparability) to follow him in this respect. However, it is to be noted that one need not be committed to the probabilistic framework in order to spell out the present account. Indeed, the proper logic with axiomatization, semantics, soundness, and completeness as presented in Huber (2007b) is formalized in terms of ranking functions (Spohn 1988, Huber 2006).

So assume a language  $\mathcal{L}$ , i.e., a set of well-formed formulas closed under negation and conjunction, and a probability measure on  $\mathcal{L}$ , i.e., a real-valued function  $\text{Pr}$  on  $\mathcal{L}$  that is non-negative, normalized, and finitely additive (i.e., such that for any sentences  $H$  and  $H'$  in  $\mathcal{L}$ :  $\text{Pr}(H) \geq 0$ ;  $\text{Pr}(H) = 1$  if  $H$  is logically valid; and  $\text{Pr}(H \vee H') = \text{Pr}(H) + \text{Pr}(H')$  whenever  $H$  and  $H'$  are not compatible). The conditional probability of  $H$  given  $E$ ,  $\text{Pr}(H|E)$ , is defined as the fraction  $\text{Pr}(H \wedge E)/\text{Pr}(E)$  whenever the probability of  $E$  is non-zero.

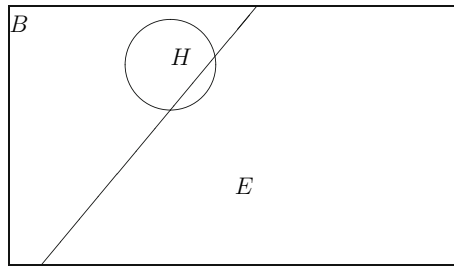
Given such a probability measure  $\text{Pr}$  on  $\mathcal{L}$ , we say for any sentences  $H$ ,  $H'$ , and  $E$  in  $\mathcal{L}$  that  $H$  is *at least as plausible as  $H'$  given  $E$*  (in the sense of  $\text{Pr}$ ) if and only if the probability of  $H$  given  $E$  is at least as great as the probability of  $H'$  given  $E$ , i.e.,  $\text{Pr}(H|E) \geq \text{Pr}(H'|E)$ , provided the probability of  $E$  is non-zero. If the probability of  $E$  equals 0, the relation is not defined, though we could equally well stipulate that in this case  $H$  is at least as plausible as  $H'$  given  $E$ .

Furthermore we say that  $H$  *informs us at least as much about  $E$  as does  $H'$*  if and only if the probability of  $\neg H$  given  $\neg E$  is at least as great as the probability of  $\neg H'$  given  $\neg E$ , i.e.,  $\text{Pr}(\neg H|\neg E) \geq \text{Pr}(\neg H'|\neg E)$ , provided the probability of  $\neg E$  is non-zero. As before, this relation is not defined if the probability of  $\neg E$  equals 0, though we could equally well stipulate that in this case any  $H$  informs us at least as much about  $E$  as any  $H'$ .  $\text{Pr}(\neg H|\neg E)$  is Hempel's generalized measure of systematic power  $s$  (Hempel and Oppenheim 1948: 164–173, esp. 172).

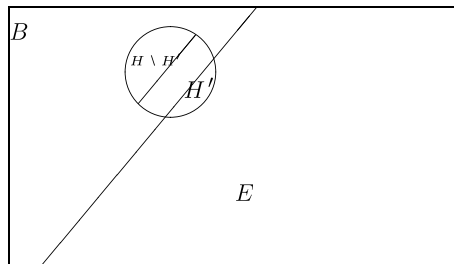
Finally, we say that  $H$  is *at least as informative as  $H'$  (independently of  $E$ )* if and only if the probability of  $\neg H$  is at least as great as the probability of  $\neg H'$ , i.e.,  $\text{Pr}(\neg H) \geq \text{Pr}(\neg H')$ .

$\Pr(\neg H)$  is Hempel's content measure  $g$  (Hempel and Oppenheim 1948: 164–173, esp. 171–172), in terms of which he defines  $s$  as  $s(H, E) = g(H \vee E)/g(E)$ .

On an epistemic interpretation of probability (whether subjective or logical does not matter), the first definition hardly needs any discussion. As to the second definition, the idea is that  $\Pr(\neg H|\neg E)$  reflects how much  $H$  informs us about  $E$ . Consider the following figure with hypothesis  $H$  and evidence  $E$  ( $B$  is the background information), all conceived of as propositions (sets of possibilities).



Suppose you are given  $E$  and then asked to strengthen the proposition  $H$  by deleting possibilities verifying it, that is, by shrinking the area representing  $H$ . Would you not delete possibilities outside  $E$ ? After all, given  $E$ , those are exactly the possibilities known not to be the actual one, whereas the possibilities inside  $E$  are still live options. This is exactly what is captured by  $\Pr(\neg H|\neg E)$ , and hence it increases when  $H$  shrinks to  $H'$  as depicted below. For an axiomatic justification see Hilpinen (1970).



A consequence of adopting  $\Pr(\neg H|\neg E)$  as one's measure of informativeness about the data is that any two hypotheses that logically imply the data are equally—indeed, maximally—informative about the data. In order to avoid this one can consider the data-independent informativeness of  $H$ , measured by  $\Pr(\neg H)$ . For  $\Pr(\neg H)$ , it does not matter whether one deletes possibilities inside or outside  $E$  (provided they have equal weight on  $\Pr$ ). The third option of considering how much the information in  $H$  goes beyond that provided by  $E$ , measured by  $\Pr(\neg H|E)$  and considered, for instance, by Carnap and Bar-Hillel (1952), requires one to delete the possibilities inside  $E$ . It is inadequate in the present context (which does not mean that it is not adequate in other contexts).

Although the logic will be different when based on the one or the other notion of informativeness, for the present purposes it does not matter which notion we take. In fact, it suffices if we base the comparison on any weighted mixture of  $\Pr(\neg H|\neg E)$  and  $\Pr(\neg H)$  (indeed, any function of the two will do which is non-decreasing in both arguments and increasing in at least one). In sum, for any language  $\mathcal{L}$  and any probability measure  $\Pr$  on

$\mathcal{L}$ , we say that  $H$  is an *acceptable hypothesis for  $E$*  in the weakly/strongly data-dependent sense (or any other sense in between),  $E| \sim_{\text{Pr}} H$  and  $E| \approx_{\text{Pr}} H$ , respectively, if and only if  $H$  is at least as plausible given  $E$  as its negation and  $H$  is more informative (about  $E$ ) than its negation, or  $H$  is more plausible given  $E$  than its negation and at least as informative (about  $E$ ) as its negation. That is,

$$\begin{aligned} & \Pr(H|E) \geq \Pr(\neg H|E) \ \& \ \Pr(\neg H) > \Pr(H), \quad \text{or} \\ & \Pr(H|E) > \Pr(\neg H|E) \ \& \ \Pr(\neg H) \geq \Pr(H) \end{aligned}$$

and

$$\begin{aligned} & \Pr(H|E) \geq \Pr(\neg H|E) \ \& \ \Pr(\neg H|\neg E) > \Pr(H|\neg E), \quad \text{or} \\ & \Pr(H|E) > \Pr(\neg H|E) \ \& \ \Pr(\neg H|\neg E) \geq \Pr(H|\neg E), \end{aligned}$$

respectively.  $| \sim_{\text{Pr}}$  and  $| \approx_{\text{Pr}}$  are the weakly/strongly data-dependent assessment relations induced by the probability measure  $\text{Pr}$  on the language  $\mathcal{L}$ .

Logical relations, such as consequence relations or relations like the two just mentioned, are usually characterized both semantically (as above) and syntactically by a list of axioms and rules that characterize the relation completely, relative to the semantic characterization. This is done in Huber (2007b) for the rank-theoretic version of strongly data-dependent assessment relations.

Another way of combining plausibility and informativeness is to consider the *expected informativeness* of hypothesis  $H$  in relation to evidence  $E$  and background information  $B$ . In this case we have to think of the informativeness of  $H$  (in relation to  $E$  and  $B$ ) as a random variable  $I$  that takes on one value, say  $i^+$ , if  $H$  is true, and another value, say  $i^-$ , if  $H$  is false. These two values  $i^+$  and  $i^-$  of the random variable  $I$  are then weighted by the probabilities (given  $E$  and  $B$ ) that  $H$  is true and false, respectively. This idea is pursued by Hempel (1960), Hintikka and Pietarinen (1966), and Levi (1961; 1963; 1967). I discuss these approaches in Huber (2007a).

## 6 Carnap's analysis revisited

In conclusion, let us turn back to Carnap's analysis of Hempel's conditions and his claim that Hempel was mixing up the explicanda underlying absolute and incremental confirmation. As argued in the previous sections, Carnap's analysis is neither charitable nor illuminating, and there is a more charitable interpretation that is illuminating by accounting for Hempel's triviality result and his rejection of the Converse Consequence Condition. Still, one might be interested in the relation between Carnap's favored concept of qualitative confirmation—viz. positive probabilistic relevance in the sense of a regular probability measure  $\text{Pr}$ —and our assessment relations leading to sufficiently plausible and sufficiently informative hypotheses.

Given the same probabilistic framework, it is clear that positive relevance of  $E$  for  $H$  is a necessary condition for  $H$  to be an acceptable hypothesis for  $E$ . More precisely, we have for any probability measure  $\text{Pr}^3$ , and any sentences  $H$  and  $E$ :

<sup>3</sup> The same holds true for any ranking function and the corresponding notion of positive rank-theoretic relevance.



$$E| \sim_{\text{Pr}} H \text{ or } E| \approx_{\text{Pr}} H \Rightarrow \Pr(H|E) > \Pr(H).$$

However, the converse is not true. Both probabilistic and rank-theoretic positive relevance are symmetric, whereas assessment relations are not.

**Acknowledgements** This research was supported by the Ahmanson Foundation as well as by the Alexander von Humboldt Foundation, the Federal Ministry of Education and Research, and the Program for the Investment in the Future (ZIP) of the German Government through a Sofja Kovalevskaja Award, while I was a member of the *Philosophy, Probability, and Modeling* group at the Center for Junior Research Fellows at the University of Konstanz.

## References

- Carnap, R. (1962). *Logical foundations of probability* (2nd ed.). Chicago: University of Chicago Press.
- Carnap, R., & Bar-Hillel, Y. (1952). *An Outline of a Theory of Semantic Information*. Technical Report No. 247 of the Research Laboratory of Electronics, Massachusetts Institute of Technology. Reprinted in Y. Bar-Hillel (1964), *Language and information. Selected essays on their theory and application* (pp. 221–274). Reading, MA: Addison-Wesley.
- Fitelson, B. (1999). The plurality of bayesian measures of confirmation and the problem of measure sensitivity. *Philosophy of Science*, 66, S362–S378.
- Flach, P. A. (2000). Logical characterisations of inductive learning. In D. M. Gabbay & R. Kruse (Eds.), *Abductive reasoning and learning* (pp. 155–196). Dordrecht: Kluwer Academic Publishers.
- Hempel, C. G. (1943). A purely syntactical definition of confirmation. *Journal of Symbolic Logic*, 8, 122–143.
- Hempel, C. G. (1945). Studies in the logic of confirmation. *Mind*, 54, 1–26, 97–121. Reprinted in C. G. Hempel (1965), *Aspects of scientific explanation and other essays in the philosophy of science* (pp. 3–51). New York: The Free Press.
- Hempel, C. G. (1960). Inductive inconsistencies. *Synthese*, 12, 439–469. Reprinted in C.G. Hempel (1965), *Aspects of scientific explanation and other essays in the philosophy of science* (pp. 53–79). New York: The Free Press.
- Hempel, C. G., & Oppenheim, P. (1948). Studies in the logic of explanation. *Philosophy of science* 15, 135–175. Reprinted in C. G. Hempel (1965), *Aspects of scientific explanation and other essays in the philosophy of science* (245–290). New York: The Free Press.
- Hintikka, J., & Pietarinen, J. (1966). Semantic information and inductive logic. In J. Hintikka & P. Suppes (Eds.), *Aspects of inductive logic* (pp. 96–112). Amsterdam: North-Holland.
- Hilpinen, R. (1970). On the information provided by observations. In J. Hintikka & P. Suppes (Eds.), *Information and inference* (pp. 97–112). Dordrecht: D. Reidel.
- Huber, F. (2006). Ranking functions and rankings on languages. *Artificial Intelligence*, 170, 462–471.
- Huber, F. (2007a). Assessing theories, bayes style. *Synthese*, doi: [10.1007/s11229-006-9141-x](https://doi.org/10.1007/s11229-006-9141-x)
- Huber, F. (2007b). The logic of theory assessment. *Journal of Philosophical Logic*, doi: [10.1007/s10992-006-9044-9](https://doi.org/10.1007/s10992-006-9044-9)
- Levi, I. (1961). Decision theory and confirmation. *Journal of Philosophy*, 58, 614–625.
- Levi, I. (1963). Corroboration and rules of acceptance. *British Journal for the Philosophy of Science*, 13, 307–313.
- Levi, I. (1967). *Gambling with truth. An essay on induction and the aims of science*. New York: Knopf.
- Milne, P. (2000). Is there a logic of confirmation transfer? *Erkenntnis*, 53, 309–335.
- Spohn, W. (1988). Ordinal conditional functions: A dynamic theory of epistemic states. In W. L. Harper & B. Skyrms (Eds.), *Causation in decision, belief change, and statistics II* (pp. 105–134). Dordrecht: Kluwer.
- Zwirn, D., & Zwirn, H. P. (1996). Metaconfirmation. *Theory and Decision*, 41, 195–228.