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Why follow the royal rule?

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Abstract This note is a sequel to Huber (Synthese 191:2167–2193, 2014). It is shown that obeying a normative principle relating counterfactual conditionals and conditional beliefs, viz. the royal rule, is a necessary and sufficient means to attaining a cognitive end that relates true beliefs in purely factual, non-modal propositions and true beliefs in purely modal propositions. Along the way I will sketch my idealism about alethic or metaphysical modality.

Keywords Counterfactuals · Conditional belief · Modal idealism · Ranking functions · Royal rule · Tracking

1 Introduction

Huber (2014) proposes a normative principle, the royal rule, that relates counterfactual conditionals and conditional beliefs. According to this principle an ideal doxastic agent's grade of disbelief in a proposition A conditional on the assumption that the counterfactual distance to the closest A-worlds equals n, and no further information that is inadmissible, ought to be equal to n. Huber (2014) stresses frequently that the royal rule is not to be accepted on intuitive grounds, but because it is a means to attaining some pertinent cognitive end. Yet Huber (2014) leaves us in the dark as to what this pertinent cognitive end might be. The present note aims to fill this lacuna by showing that the royal rule is a necessary and sufficient means to attaining a cognitive goal that can be thought of as a counterfactual variant (in terms of specific conditional

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beliefs that will be discussed below) of what James (1896) calls "our first and great commandments as would-be knowers", namely:

"Believe truth! Shun error!"

This counterfactual variant has its origin in Nozick's (1981) sensitivity and adherence conditions, although these conditions, in contrast to the end described below, are formulated in terms of non-conditional belief. Ignoring details that do not matter for the present paper (such as the method that is used in the belief formation), Nozick's (1981) conditions say:

Sensitivity For any proposition A: If A were false, then the ideal doxastic agent would not believe A.

Adherence For any proposition A: If A were true, then the ideal doxastic agent would believe A.

Nozick (1981) intends these two conditions as bridging the gap between true belief on the one hand and knowledge on the other hand (for a recent discussion see Roush 2005). Sosa (1999) and Williamson (2000) think a different condition, safety, is better suited for describing a distinctive feature of knowledge. This latter condition says:

Safety For any proposition A: If the ideal doxastic agent believed A, then A would be true.

Conceptual analysis is not the business of the present author, nor is explication, and little is less relevant to the present note than alleged counterexamples to Nozick's (1981) analysis of knowledge on the basis of subjective intuitions.

Nor is this note intended to suggest that there would be anything *objectively* desirable about sensitivity or adherence or safety in any of their formulations, conditional or otherwise. Instead, the aim is the modest one of establishing a means-end relationship between a normative principle on the one hand and a pertinent cognitive end on the other hand.

First the royal rule will be shown to be sufficient for a weak and conditional formulation of sensitivity. (It will also be shown to be sufficient for a weak and conditional formulation of adherence given an assumption about the logic of counterfactuals that the present paper does not make.) Then the royal rule will be shown to be necessary and sufficient for a strong and conditional formulation of sensitivity. These conditional formulations resemble Nozick's (1981) conditions in important respects, which is why I term them accordingly. However, they also differ from Nozick's (1981) conditions in equally important respects. In particular, the present conditions differ from Nozick's (1981) by being formulated in terms of specific conditional beliefs. This divergence away from non-conditional belief, towards specific conditional beliefs, will be motivated by a discussion of "modally agnostic" agents. Specifically, the present conditions are not intended to be formalizations or improvements of Nozick's (1981) conditions, but plain and simple cognitive ends of independent interest that an ideal doxastic agent may or may not have.

The structure of this note is as follows. Section 2 describes the relevant theory of conditional beliefs. Sections 3 and 4 present the semantics for counterfactuals and the royal rule that relates counterfactual conditionals and conditional beliefs. Readers who are familiar with these items can directly move to Sect. 5 which contains the argument for the thesis that the royal rule is a necessary and sufficient means to attaining the cognitive end of tracking the facts in the sense of a strong, but conditional version of sensitivity.

The framework adopted for the present paper is Huber's (2014). The latter distinguishes between the factual component and the (alethically) modal component of a possible world. As will be seen, this leads to complications that prevent the study of safety. Therefore in Appendix I will sketch how one might study which normative principles one has to obey in order to attain the cognitive ends described by various formulations of safety. In addition I will indicate how a Bayesian might study what cognitive ends Lewis' (1980) principal principle is a means to attaining.

2 Conditional belief

Ranking functions have been introduced by Spohn (1988, 2012) in order to model qualitative conditional belief. Ranking theory is quantitative or numerical in the sense that ranking functions assign numbers to propositions, which are the objects of belief in this theory. These numbers are needed for the definition of conditional ranking functions representing conditional beliefs. As we will see, though, once conditional ranking functions are defined we can interpret everything in purely qualitative, but conditional terms.

Consider a set of possible worlds W and an algebra of propositions $\mathcal{A}$ over W. A function $\varrho : \mathcal{A} \to \mathbb{N} \cup \{\infty\}$ from $\mathcal{A}$ into the set of natural numbers $\mathbb{N}$ extended by ∞, $\mathbb{N} \cup \{\infty\}$, is a *finitely/countably/completely minimitive ranking function* on $\mathcal{A}$ just in case for all finite/countable/arbitrary sets of propositions $\mathcal{B} \subseteq \mathcal{A}$:

$$\varrho(W) = 0 \tag{1}$$

$$\varrho(\emptyset) = \infty \tag{2}$$

$$\varrho\left(\bigcup \mathcal{B}\right) = \min\{\varrho(A) : A \in \mathcal{B}\} \tag{3}$$

For a non-empty or consistent proposition $A \neq \emptyset$ from $\mathcal{A}$ the conditional ranking function $\varrho(\cdot \mid A) : \mathcal{A} \setminus \{\emptyset\} \to \mathbb{N} \cup \{\infty\}$ based on the unconditional ranking function $\varrho(\cdot) : \mathcal{A} \to \mathbb{N} \cup \{\infty\}$ is defined as

$$\varrho(\cdot \mid A) = \begin{cases} \varrho(\cdot \cap A) - \varrho(A), & \text{if} \quad \varrho(A) < \infty, \\ \infty \text{ or } 0, & \text{if} \quad \varrho(A) = \infty. \end{cases}$$

For the case where $\varrho(A) = \infty$ Goldszmidt and Pearl (1996, p. 63) suggest ∞ as value for $\varrho(B \mid A)$ for all $B \in \mathcal{A}$. For this case Huber (2006, p. 464) suggests 0 as value for $\varrho(B \mid A)$ for all non-empty $B \in \mathcal{A}$ and additionally stipulates $\varrho(\emptyset \mid A) = \infty$ to ensure that every conditional ranking function is a ranking function on $\mathcal{A}$.

A ranking function ϱ is *regular* if and only if

$$\varrho(A) < \varrho(\emptyset) \tag{4}$$

for all non-empty or consistent propositions A from $\mathcal{A}$. In contrast to probability theory it is always possible to define a regular ranking function, no matter how rich or fine-grained the underlying algebra of propositions.

Doxastically ranks are interpreted as grades of disbelief. A proposition A is *disbelieved* just in case A is assigned a positive rank, $\varrho(A) > 0$. A is *believed* just in case its complement or negation, $\overline{A}$, is disbelieved, $\varrho\left(\overline{A}\right) > 0$.

A proposition A is *disbelieved conditional on* a proposition C just in case A is assigned a positive rank conditional on C, $\varrho(A \mid C) > 0$. A is *believed conditional on* C just in case its complement or negation, $\overline{A}$, is disbelieved conditional on C, $\varrho\left(\overline{A} \mid C\right) > 0$. It takes getting used to reading positive numbers in this "negative" way, but mathematically this is the simplest formulation of ranking theory. Note that a proposition A is believed just in case A is believed conditional on the tautological proposition W. This is so, because $\varrho\left(\overline{A}\right) = \varrho\left(\overline{A} \mid W\right)$.

It follows from the definition of conditional ranking functions that the ideal doxastic agent should not disbelieve a non-empty or consistent proposition A conditional on itself: $\varrho(A \mid A) = \varrho(A \cap A) - \varrho(A) = 0$. I'll refer to this consequence below. In doxastic terms the first axiom says that the ideal doxastic agent should not disbelieve the tautological proposition W. The second axiom says that she should disbelieve the empty or contradictory proposition $\emptyset$ with maximal strength ∞. Given the definition of conditional ranking functions, the second axiom can be read in purely qualitative, but conditional terms. Read this way it says that the ideal doxastic agent should disbelieve the empty or contradictory proposition conditional on any proposition with a finite rank. This implies that she should believe the tautological proposition with maximal strength, or conditional on any proposition with a finite rank.

Finite minimitivity is the weakest version of the third axiom. It states that $\varrho(A \cup B) = \min\{\varrho(A), \varrho(B)\}$ for any two propositions A and B in the algebra. Part of what finite minimitivity says is that the ideal doxastic agent should disbelieve a disjunction $A \cup B$ just in case she disbelieves both its disjuncts A and B. Given the definition of conditional ranking functions, finite minimitivity extends this requirement to conditional beliefs. As noted above the definition of conditional ranking functions implies that the ideal doxastic agent should not disbelieve a proposition conditional on itself. Given this consequence, finite minimitivity says the following (in purely qualitative, but conditional terms): the ideal doxastic agent should conditionally disbelieve a disjunction $A \cup B$ just in case she conditionally disbelieves both its disjuncts A and B. Countable and complete minimitivity extend this requirement to disjunctions of countably and arbitrarily many disjuncts, respectively.

Interpreted doxastically these axioms are synchronic norms for organizing the ideal doxastic agent's beliefs and conditional beliefs at a given moment in time. They are supplemented by three diachronic norms for updating her beliefs over time if new information of various formats is received.

Plain conditionalization (Spohn 1988) mirrors the update rule of strict conditionalization from probability theory (Vineberg 2000): it is defined for the case where the new information comes in form of a "certainty," a proposition that the ideal doxastic agent comes to believe with maximal strength. Spohn conditionalization (Spohn 1988) mirrors the update rule of Jeffrey conditionalization from probability theory (Jeffrey 1983): it is defined for the case where the new information comes in form of new ranks for the elements of an "evidential partition." Shenoy conditionalization (Shenoy 1991) mirrors the update rule of Field conditionalization from probability theory (Field 1978): it is defined for the case where the new information reports the differences between the old and the new ranks for the elements of an evidential partition.

The resulting package of synchronic and diachronic norms can be justified by the consistency argument (Huber 2007) in much the same way that probability theory can be justified by the Dutch book argument. The consistency argument shows that obeying the synchronic and diachronic rules of ranking theory is a necessary and sufficient means to attaining the cognitive end of always holding beliefs that are jointly consistent and deductively closed. To the extent that the ideal doxastic agent has this goal, she should obey the norms of ranking theory. It is not that we are telling her what and how to believe. *She* is the one who has this goal. We merely point out the objectively obtaining means-ends relationships. Of course, if the ideal doxastic agent does not aim at always holding beliefs that are jointly consistent and deductively closed, our response will cut no ice. But that is besides the point: it is mistaking a hypothetical imperative for a categorical one.

3 Counterfactual conditionals and modal idealism

There are at least two sorts of conditionals (Adams 1970): indicative conditionals and counterfactual, or subjunctive, conditionals.[1] Each can be approached in at least two ways. On the one view conditionals express propositions that are true or false (Stalnaker 1968; Lewis 1973a). On the other view conditionals do not express propositions that are true or false. On this latter view conditionals do not have truth values, but express the uttering or thinking agent's conditional state of mind rather than a state of affairs that may or may not obtain in the external reality. The latter view is prominent for indicative conditionals (Adams 1975), but some also adopt it for counterfactual conditionals (Edgington 2008; Spohn 2013, 2015).

This paper is concerned with counterfactual conditionals and assumes that they express propositions that are true or false. Indicative conditionals will only play a role insofar as they express conditional beliefs. While I think this is the right view of indicative conditionals, nothing hinges on this assumption: if it was false, I would just not be saying anything about indicative conditionals.

The assumption that counterfactual conditionals express propositions that are true or false is grave, though. Yet it does not bring with it what is known as "modal realism" (Lewis 1986a). The view adopted here is as simple as it is perhaps naïve. A modal language or system of representation allows one to say *how* something is the case: it could be the case that it rains, the streets would be wet if it did, and chances are that one slips if the streets are wet. A factual or non-modal language allows one to say *what* is the case without allowing one to say how it is the case: it rains, the streets are wet, one slips. For each factual language or system of representation there is at most one linguistic or conceptual or representational entity that accurately and maximally specifically, i.e. as completely as the factual language or system of representation

[1] Not everybody is willing to identify the latter two, though. See Bennett (2003).

allows, describes or represents reality. Let us call this linguistic or conceptual or representational entity *the actual factual "world"* for the factual language or system of representation under consideration.

The actual factual world is not real. To use a dangerously loaded term, it is an *idea*, a mind-dependent construct that is similar to a state description in Carnap's (1947) sense. Besides the actual factual world there are many merely possible factual worlds, i.e. descriptions or representations that maximally specifically, but inaccurately describe or represent reality. These factual worlds include every description or representation the syntactic machinery of the factual language or system of representation allows for. (In principle these descriptions or representations may include ones that are not maximally specific, as well as ones which are too specific in the sense that they can be disproved in some proof system for this language or system of representation; see Huber (2015a). However, for the present paper we can ignore such incomplete or impossible factual worlds.)

These factual worlds gives rise to factual propositions, which we can formally represent as sets of factual worlds. So much for a factual language.

In a modal language we can say more than in a factual language. In addition to being able to say that it does not rain, we can say that it could have rained, and that the streets would have been wet if it did, and that chances are that one slips if it does. These modal claims translate into claims about the existence of some factual worlds, about the counterfactual distance of some factual propositions to some factual worlds, and about the chances of some factual propositions at some factual worlds.

The modality relevant for the present paper is counterfactuality. That is, we are concerned with claims about relative counterfactual distances that are expressed by counterfactual conditionals. Like the notion of a factual world, the notion of counterfactual distance is relative to a language or system of representation (in this case, a modal language). On my view the same is true of chance, but this is not our topic. Neither worlds nor propositions nor counterfactual distances are real in any language- or thought- or other representation-independent sense. We talk about and think about and conceptualize and represent reality in terms of what is and what is not, in terms what could have been, and in terms of what would have been. And we do so, because we find it useful. Yet these *not*s and *could*s and *would*s are not part of reality. They belong to the language or system of representation we use to describe or represent reality, and thus to the realm of the mental. As its name suggests, the only thing that is really real is reality.

It is true that it did not rain or snow, but that it could have rained, and that the streets would have been wet if it had rained. On the present view, there is nothing in reality that is described by, or corresponds to, or makes true, these *not*s and *or*s and *and*s and *could*s and *would*s. They are tied to our representation of reality in thought and language—and needless to say, we cannot think, let alone talk, about reality without some representation. Just as thinking and talking about reality are dependent on a system of representation or language, so is truth.

This is why these claims and thoughts can have truth values without there being anything in reality that makes them true. What is true, and what is not true, and what could have been true and what would have been true, depend on reality, because the actual factual world does so depend. Yet what is true *also* depends on the language or system of representation that these propositions belong to. Counterfactual conditionals and other alethically modal claims can express truths without there being any alethically modal reality that makes them true: alethic modality is "idealistic" (though neither analytic nor a priori nor necessary).

In sum, the present view—call it *modal idealism*—is a third option between the realist view and the expressivist view. Like the expressivist, it does not locate the modalities in reality, but in the mind. Like the realist, it does not interpret the modalities as expressing beliefs, but as expressing propositions that are true or false (of course, what counts as a proposition is radically different on these two accounts). Modalities are *ideas*. However, they are not ideas with objective reality. They are *our* ideas. *We* conceptualize reality in terms of *could*s and *would*s, and we do so for the exact same reason that we conceptualize reality in terms of *not*s and *or*s and *and*s: because we find it useful. Different beings who also think or talk about reality may conceptualize it in different terms or ideas. There is no right or wrong here, just a more or less useful for various purposes.

Ideas are not beliefs, though, but their objects. The latter depend on the former, as we can only have beliefs that something is true if we have a language or system of representation that provides the objects or contents of these beliefs.

Beatrice claims that it is not the case that there are two capitals of Australia, but that there could have been; and if there had been, there would have been two prime ministers. Colin claims that there could not have been two capitals of Australia. Desiree claims that there would not have been two prime ministers of Australia, even if there had been two capitals of Australia.

Do Beatrice, Colin, and Desiree disagree or do they speak different languages or neither? Assuming they speak the same language, the expressivist—e.g. Spohn (2015)—must say that, in general, there is no disagreement between them, just a difference in conditional belief and strength of belief, yet no proposition that the one believes to be true, but the others do not believe to be true.

Assuming they speak the same language, the present—idealist—view says that there is disagreement, as does the realist. Yet whereas the realist cannot, the idealist can say that there need not be disagreement. For the realist (about possible worlds and about counterfactual distance[2]), it is reality alone that decides what could have been, and what would have been. For the idealist (about possible worlds and counterfactual distance), this decision is jointly made by reality and by the language or system of representation. The idealist can say that the three speak different languages, none of which is right or wrong, but simply more or less useful. For the realist there is an objective right or wrong that is determined by reality: the realist must say that at most one of them speaks the right language.

Still, one might wonder, why not follow the expressivist all the way? Well, the expressivist can explain, or express, away only so much. She can explain away moral talk, and she can explain away indicative-conditional talk. However, she cannot explain away all talk unless she is willing to say that there are no truths whatsoever, in which case we do not talk or think *about anything* anymore: we just talk or think, without

[2] Lewis (1986a) is a realist about possible worlds, but not about counterfactual distance (Lewis 1979). Lewis (1986b, 1994) does not want to be a realist about chance.

saying or meaning anything. Suppose Adam claims that there are two capitals of Australia. As long as we want to say that Adam says something and does not merely make meaningless sounds, the expressivist about *not* cannot explain away the disagreement between him and Beatrice.

We thus need some ideas or propositions, and the present view includes alethically modal propositions such as those expressed by counterfactual conditionals.[3]

Counterfactual conditionals express propositions about the relative counterfactual distance of various factual propositions to some factual world. They are closely tied to causation (Lewis 1973b; Collins et al. 2004; Huber 2013) and they relate to what statisticians call the *mode* of a sample much like chances relate to relative frequencies (Huber 2015b). The big question is, of course, what these counterfactual distances are like. This question is answered by the *royal rule* (Huber 2014): counterfactual distances have the structure of ranking functions. For this reason, and in order to distinguish them from subjective ranking functions representing belief and conditional belief, Huber (2014) formally represents them by what are called "objective" ranking functions. In light of the above it is perhaps better to term them "idealistic" or, more neutrally, *alethic* ranking functions, with the understanding that alethic modality is language- or representation-dependent and not part of the (objective) reality. Whatever their name, substituting these ranking functions for Stalnaker's (1968) selection functions or Lewis's (1973a) systems of spheres results in a semantics for counterfactual conditionals that is sound and complete with respect to the conditional logic **V** as follows.

Let $\mathcal{L}_0$ be the smallest set that includes a given countable set of propositional variables PV and is closed under the classical connectives '$\neg$', '$\wedge$', '$\vee$', and '$\supset$'. Let $\mathcal{L}_1$ be the smallest set containing all elements of $\mathcal{L}_0$ and $\ulcorner \alpha \mathrel{\Box\!\!\rightarrow} \beta \urcorner$ for any two well-formed formulae α and β from $\mathcal{L}_0$. Let $\mathcal{L}$ be the smallest set that includes $\mathcal{L}_1$ and is closed under the classical connectives.

Thus $\mathcal{L}$ is built up form a countable set of propositional variables in the usual way, with the only exception that $\ulcorner \alpha \mathrel{\Box\!\!\rightarrow} \beta \urcorner$ is a well-formed formula if, *and only if*, α and β are well-formed formulae *and do not contain an occurrence of* '$\Box\!\!\rightarrow$'. I will use all symbols autonymously from now on.

The following corrects Huber's (2014, p. 2190) ill-defined notion of a rank-theoretic model: $(F, \mathcal{A}_F, R, W, [\![\,]\!])$ is a *rank-theoretic model* for $\mathcal{L}$ just in case F is a non-empty set of factual worlds, $\mathcal{A}_F$ is an algebra over F, R is a set of ranking functions $r : \mathcal{A}_F \rightarrow \mathbb{N} \cup \{\infty\}$, $W \subseteq F \times R$ is such that for each $f \in F$ there is *at least one* $r \in R$ such that $(f, r) \in W$, and $[\![\,]\!] : \mathcal{L} \rightarrow \wp(W)$ is an interpretation function such that for all α and β from $\mathcal{L}$:

1. if $p \in PV$, then $[\![p]\!] \subseteq W$ is such that: if $(f, r) \in [\![p]\!]$ for some $f \in F$ and some $r \in R$, then $\left(f, r'\right) \in [\![p]\!]$ for all $r' \in R$ such that $\left(f, r'\right) \in W$; and $fact\left([\![p]\!]\right) \in \mathcal{A}_F$
2. $[\![\neg\alpha]\!] = W \setminus [\![\alpha]\!]$, $[\![\alpha \wedge \beta]\!] = [\![\alpha]\!] \cap [\![\beta]\!]$, and analogously for $\vee$ and $\supset$
3. $[\![\alpha \mathrel{\Box\!\!\rightarrow} \beta]\!] = \left\{w = (f_w, r_w) \in W : \left(fact\left([\![\alpha]\!]\right)\right)^{r_w} \subseteq fact\left([\![\beta]\!]\right)\right\}$,

[3] This choice may seem arbitrary, but the reader needs to keep in mind that the costs of an expressivist account of counterfactuals are huge. If counterfactuals do not express propositions, then the same is true of any theory relying on counterfactuals, such as counterfactual theories of causation, causal decision theory, dispositions, knowledge, responsibility etc.

where $fact([\![\alpha]\!]) = \{f \in F : \exists r \in R((f, r) \in [\![\alpha]\!])\}$ and the superscript 'r_w' indicates that we pick out those factual $[\![\alpha]\!]$-worlds that are r_w-minimal,[4]

$$A^{r_w} = \{f \in A \in \mathcal{A}_F : \forall B \in \mathcal{A}_F \text{ (if } f \in B, \text{ then } r_w(B) \leq r_w(A))\}.$$

4 The royal rule

The reason for this complicated way of setting things up is that Huber (2014) thinks it is important to distinguish between the factual components f_w and the modal components r_w of possible worlds $w = (f_w, r_w)$ about which beliefs will be formed (for an alternative set-up see the appendix). This in turn is so because of the *royal rule*, a normative principle according to which an ideal doxastic agent's conditional beliefs are constrained or guided by information about counterfactual distances. This rule assumes that counterfactual distances are numerical, but no more. The rule then implies that numerical counterfactual distances have the same structure as the agent's conditional beliefs. Since rational conditional beliefs have the structure of a ranking function, so do counterfactual distances. Thus we do not *postulate* that modal ideas or propositions have this structure. Instead, we *derive* their structure from a rationality constraint on beliefs.

For the present paper we may use this to simplify things slightly. We can model counterfactual distances by alethic ranking functions rather than first deriving this. Furthermore, we can let these alethic ranking functions r be defined on the powerset of the set of all factual worlds F, $\wp(F)$. Moreover we can let the set of possible worlds over which the agent has beliefs be $W = F \times R$, where R is the set of *all* (alethic) ranking functions on $\wp(F)$. The ideal doxastic agent's subjective ranking function ϱ, which represents her beliefs and conditional beliefs, is defined on the powerset of W, $\wp(W)$, and assumed to be regular.

Royal Rule for A**.** *Let* $\varrho : \wp(F \times R) \rightarrow \mathbb{N} \cup \{\infty\}$ *be an ideal doxastic agent's grading of disbelief, which is assumed to be a regular subjective ranking function. Let* n *be a number from* $\mathbb{N} \cup \{\infty\}$. *Let* $A \subseteq F$ *be a* fixed *factual proposition, and let '*$r(A) = n$*' denote the proposition that the counterfactual distance to the closest factual* A*-worlds equals* n, $\{w = (f_w, r_w) \in W : r_w(A) = n\}$. *Finally, let* E *be an* arbitrary *proposition that is "admissible" in the sense that it is purely modal information (if* $(f, r) \in E$ *for some* $f \in F$ *and some* $r \in R$, *then* $(f', r) \in E$ *for all* $f' \in F$*) that is consistent with* $r(A) = n$. *Then*

$$\varrho(A \times R \mid (r(A) = n) \cap E) = n.$$

The royal rule for ranks says that an ideal doxastic agent's grade of disbelief for the factual proposition $A \subseteq F$ conditional on the assumption that the counterfac-

[4] Alternatively one could pick out those factual $[\![\alpha]\!]$-worlds f that are r_w-minimal or no more distant according to r_w than f_w is (in the precise sense that is formalized in analogy to the definition of A^{r_w} above). This is an alternative definition, because f_w need not be assigned rank 0 by r_w and so can have a higher r_w-rank than the r_w-minimal factual $[\![\alpha]\!]$-worlds. In this case the resulting semantics is sound and complete with respect to the conditional logic **VW** that results from **V** by adding the axiom schema: $(\alpha \wedge (\alpha \mathrel{\Box\!\!\rightarrow} \gamma)) \supset \gamma$.

tual distance to the closest A-worlds equals n, and no further information that is not admissible, ought to be equal to n.

Ranks are numerical, but unlike probabilities, which are measured on an absolute scale, neither subjective nor alethic ranks utilize all the information carried by these numbers. Instead, subjective ranks, and hence alethic ranks, are at best measured on a ratio scale (Hild and Spohn 2008). The royal rule is thus weaker than it might at first appear: alethic ranks guide subjective ranks *provided* the former are reported in terms of the latter's scale. Otherwise the royal rule is silent, as we would be comparing apples and oranges.

It will prove useful below to adopt the following terminology. Say that *the Stalnaker assumption obtains* just in case for all factual propositions $A \subseteq F$ and all possible worlds $(f_w, r_w) = w \in W$: $r_w(A) > 0$ or $r_w(F \setminus A) > 0$. The Stalnaker-assumption validates the principle of conditional excluded middle:

$$(\alpha \;\Box\!\!\rightarrow \gamma) \vee (\alpha \;\Box\!\!\rightarrow \neg\gamma)$$

The present paper does not make the Stalnaker-assumption, because this assumption does not follow from the royal rule. However, the present paper does assume that, for all factual propositions $A \subseteq F$, at least one of A and $F \setminus A$ has alethic rank equal to 0. This assumption follows from the royal rule (Huber 2014). Lewis (1973a: 19f) terms it "the limit assumption" and rejects it.[5]

In what follows I will exclusively focus on propositions and ignore sentences.

5 The royal rule tracks the facts

Consider the following conditions.

A-Sensitivity to A For a *fixed* factual proposition $A \subseteq F$: If A were true, then the ideal doxastic agent would not disbelieve A in the sense that $\varrho(A \times R) = 0$.

A-Adherence to A For a *fixed* factual proposition $A \subseteq F$: If A were true, then the ideal doxastic agent would believe A in the sense that $\varrho((F \setminus A) \times R) > 0$.

The second occurrence of 'A' in 'A-sensitivity to A' is to indicate that we are interested in the question whether the ideal doxastic agent's belief in A is sensitive to the truth of A. The first occurrence of 'A' in 'A-sensitivity to A' is to indicate an additional aspect in which this question will turn out to depend on A. The notion of sensitivity is formulated in terms of a counterfactual conditional. The meaning of this counterfactual conditional depends on a ranking function. As we will see below, this ranking function is not one and the same for each antecedent A. Instead there will be different ranking functions for different antecedents. To stress this additional dependence on A I will sometimes speak of the 'A-sensitivity to A', even though we will not be concerned with cases of A-sensitivity to B for some proposition B that differs from A.

[5] Herzberger (1979) shows the limit assumption to be equivalent to the condition that the set of counterfactual consequences $\{\gamma \in \mathcal{L} : \alpha \;\Box\!\!\rightarrow \gamma\}$ of any consistent sentence α be consistent.

I will first show that *certain* subjective ranking functions ϱ attain the cognitive end described by A-sensitivity to A, but fail to attain the cognitive end described by A-adherence to A, even if we make the Stalnaker assumption and assume the royal rule to be satisfied. Next I will show that *any* subjective ranking function ϱ that obeys the royal rule attains a conditional version of the former cognitive end (and the latter, if we make the Stalnaker assumption). Then I will reformulate this version in order to motivate a final and conditional version of sensitivity that will be the official formulation of tracking a fact A. I will show that, for a fixed factual proposition A, this cognitive end is attained for A by all and only these subjective ranking functions ϱ that obey the royal rule for A. This will conclude my argument for the thesis that obeying the royal rule is a necessary and sufficient means to attaining the cognitive end of tracking the facts in a strong, but conditional sense.

Consider a non-empty or consistent factual proposition $A \subseteq F$ and its complement $F \backslash A$. There are exactly two possibilities: A is true and $F \backslash A$ is false, or it is the other way round. For each of these two possibilities there are exactly three possibilities: both A and $F \backslash A$ have alethic rank equal to 0; A has alethic rank equal to 0, and $F \backslash A$ has positive alethic rank; A has positive alethic rank, and $F \backslash A$ has alethic rank equal to 0. This leaves us with six possibilities so far.

For each of these six possibilities there are exactly three possibilities: neither A nor $F \backslash A$ is believed; F is not believed, and $F \backslash A$ is believed; A is believed, and $F \backslash A$ is not believed. More rigorously, this means that ϱ does, or does not, assign a positive subjective rank to the propositions $(F \backslash A) \times R$ and $A \times R$, respectively. We thus end up with a total of 18 classes of cases to consider. (There are, of course, infinitely many cases in each of these classes, but they only differ from each other in ways that are not relevant for present purposes.)

I will first make the assumption that the ideal doxastic agent is *modally agnostic* and suspends judgment with respect to whether or not the alethic rank of any contingent factual proposition A is positive or not: $\varrho\,(r\,(A) = m) = 0$ and $\varrho\,(r\,(F \backslash A) = m) = 0$ for all numbers m from $\mathbb{N} \cup \{\infty\}$. More generally, an ideal doxastic agent is defined to be modally agnostic if and only if $\varrho\,(F \times \{r\}) = 0$ for each alethic ranking function $r \in R$.

Ideal doxastic agents that are modally agnostic and obey the royal rule are possible. They assign subjective rank 0 to $(A \times R) \cap (r\,(A) = 0) \cap (r\,(F \backslash A) = m)$ and to $((F \backslash A) \times R) \cap (r\,(A) = m) \cap (r\,(F \backslash A) = 0)$, for all $m \in \mathbb{N} \cup \{\infty\}$. In addition they assign subjective rank m to $((F \backslash A) \times R) \cap (r\,(A) = 0) \cap (r\,(F \backslash A) = m)$ and to $(A \times R) \cap (r\,(A) = m) \cap (r\,(F \backslash A) = 0)$. The assumption that the ideal doxastic agent is modally agnostic simplifies the situation. However, its purpose is to illustrate the shortcomings of the non-conditional nature of A-sensitivity to A, and of A-adherence to A, and to explain what the royal rule requires. It will be dropped as soon as we discuss the conditional formulations of sensitivity and adherence and the official definition of tracking the facts.

What the royal rule asks of modally agnostic agents can perhaps be explained with analogy to Moore's (1942) paradox. The latter is exemplified by sentences of the form 'A and I don't believe that A' or, better suited for our purposes:

A and I disbelieve that A.

Moorean sentences are logically consistent, but somehow odd to be believed or asserted. On the present picture (dis-)beliefs come in grades, which leads us to graded versions of Moorean sentences:

> A and I disbelieve A with strength n.

The oddity of such graded Moorean sentences increases with n. Minimal oddity for $n = 0$, which just means: A and I don't disbelieve A. Some oddity for $n > 0$, which means: A and I believe that A is false with strength $n > 0$. Maximal oddity for $n = \infty$, which just means: A and I am certain that A is false.

The last step then is to replace subjective grades of disbelief with counterfactual distances or "grades of bizarreness": A and the alethic rank of A equals n. Or:

> A and the bizarreness of, or counterfactual distance to, A equals n. That is, if A was true, this would be bizarre to grade n.

Such counterfactual versions of graded Moorean sentences are logically consistent unless one made Lewis (1973a, p. 14f)'s weak or strong centering assumption, which the present paper does not make.[6] They may, or may not, be odd. However, they should be disbelieved according to the royal rule. And they should be disbelieved the firmer, the higher the grade n. What the royal rule asks of modally agnostic agents is that the conjunction 'A and $r(A) = n$' be disbelieved to grade n. Ideal doxastic agents that are not modally agnostic are additionally asked to add to n their grade of disbelief that $r(A) = n$: 'A and $r(A) = n$' be disbelieved to grade $n + k$, where k is the ideal doxastic agent's grade of disbelief that $r(A) = n$.[7]

Let us return to the 18 classes of cases described above. A case ω consists of a factual component f_ω specifying the truth values of all factual propositions, an alethically modal component (the alethic ranking function) r_ω specifying the truth values of all counterfactual conditionals, and a doxastically modal component (the subjective ranking function) ϱ_ω specifying all the ideal doxastic agent's beliefs and conditional beliefs. It can be represented as $\omega = (f_\omega, r_\omega, \varrho_\omega)$ or, stressing the relevant details, as:

$$\omega = ([\![A]\!]_\omega, r_\omega(A), r_\omega(F \backslash A), \varrho_\omega(A \times R), \varrho_\omega((F \backslash A) \times R))$$

In order to make sense of A-sensitivity to A, and A-adherence to A, we now need to assign counterfactual distances or ranks to these cases ω. Otherwise the English counterfactuals in the formulation of A-sensitivity to A, and A-adherence to A, cannot be assigned a truth value.

[6] The weak centering assumption validates the axiom schema: $(\alpha \wedge (\alpha \Box\!\!\rightarrow \gamma)) \supset \gamma$. As noted in footnote 4, this axiom schema can also be validated by changing the truth-condition for the counterfactual conditional instead of making the weak centering assumption. The strong centering assumption validates the axiom schema: $(\alpha \wedge \gamma) \supset (\alpha \Box\!\!\rightarrow \gamma)$. This axiom schema cannot be validated by changing the truth-condition as in footnote 4.

[7] In classical Moorean sentences this additional part is a meta-disbelief about one's first-order disbelief in A. If made explicit it gives rise to infinitely long Moorean sentences: A and I don't believe that A and I don't believe that I don't believe that A etc. The auto-epistemological reflection principle (Spohn 2012, Chap. 9, relying on Hild (1998)) implies that the ideal doxastic agent is certain of what her own beliefs are. Therefore it rules that all these Moorean sentences be disbelieved, including the original one.

This is a tricky task, as there is the threat that I am smuggling into these counterfactual distances or ranks whatever it is that I want to derive. So let me try to be as clear as possible. The question we face can be stated as follows. We are given a set of possible cases $\Omega = F \times R \times S$, where F is the set of all possible factual components, R is the set of all alethically modal components (aka alethic ranking functions defined on the power-set of F), and S is the set of all doxastically modal components (aka subjective ranking functions defined on the power-set of $F \times R$). Our task is to define one or more ranking function(s) $rank$ on the power-set of $F \times R \times S$.

The standard move, to be sketched in the appendix, is to equip each case ω with its own ranking function $rank_\omega$. This results in a model of the form: $\left(\Omega, (rank_\omega)_{\omega \in \Omega}\right)$. However, this would run counter to the very point of Huber's (2014) set-up, which is designed to avoid that modalities can automatically be iterated indefinitely. Yet this is the case for a model of such form. The present paper does not want to defend or criticize this thought and the resulting set-up, but merely adopt it in order to fill the lacuna left in Huber's (2014) paper as explained in the introduction. For this reason a different move is mandated.

The antecedents of the counterfactual conditionals we are considering are all restricted to purely factual propositions. These purely factual propositions are assigned alethic ranks in each possible case ω, but the cases themselves are not. Yet in order to evaluate the English counterfactuals what we need are precisely ranks for the cases themselves, not just their factual components. Therefore we must assign ranks to the cases *vicariously*.

One option is to take $rank_\omega\left(\left\{\omega'\right\}\right) = r_\omega\left(\{f_{\omega'}\}\right)$. However, this is instantiating the problem mentioned above rather than avoiding it, because now each ω is assigned a ranking function $rank_\omega$ defined on the power-set of Ω. A different option is to take $rank\left(\{\omega\}\right) = r_\omega\left(\{f_\omega\}\right)$. This avoids the problem mentioned above and tells us how bizarre ω is qua f_ω-case, namely bizarre to grade $r_\omega\left(\{f_\omega\}\right)$. However, it does not tell us how bizarre ω is qua A-case, for an arbitrary factual proposition $A \subseteq F$ (unless one were to identify the bizarreness of ω qua A-case with its bizarreness qua f_ω-case, for all factual propositions $A \subseteq F$). This information is not provided by $r_\omega\left(\{f_\omega\}\right)$. It is only provided by a case ω's alethic ranking function r_ω *in its entirety*, that is, by the alethic ranks $r_\omega(A)$ for *all* factual propositions $A \subseteq F$, not just the particular factual proposition $\{f_\omega\}$. The third option is to make this idea more precise, but to stay as neutral as possible and only use as much information from r_ω as is needed in order to evaluate a counterfactual conditional.

I will take the third option. A consequence will be that the ranks of a case depend on the antecedent of the counterfactual conditional that is evaluated. That is, we will end up with a model of the form: $\left(\Omega, (rank_A)_{A \subseteq F}\right)$, where $\Omega = F \times R \times S$ and where, for each purely factual proposition $A \subseteq F$, $rank_A$ is a ranking function on the power-set of Ω that specifies how bizarre or counterfactually distant a case ω is qua case in which A is true.[8]

Suppose, then, two cases ω_1 and ω_2 agree that the factual proposition A is true. Under this assumption, the question we need to answer is this: is $\{\omega_1\}$ less bizarre or

[8] The idea that counterfactual distances are relative to the antecedent of the counterfactual conditional under evaluation is endorsed by Bigaj (2006) and, in a different context, criticized by Cross (2008).

counterfactually less distant qua A-world than $\{\omega_2\}$ in the sense of what I will call the "actual ranking function" $rank_A$? I propose that this be the case if and only if the (alethically) modal components of ω_1 and ω_2 say so of A:

$$[\![A]\!]_{\omega_1} = [\![A]\!]_{\omega_2} = true \Rightarrow \left[rank_A\left(\{\omega_1\}\right) < rank_A\left(\{\omega_2\}\right) \Leftrightarrow r_{\omega_1}(A) < r_{\omega_2}(A)\right]$$

This is possible. For instance, we can let the actual rank of $\{\omega\}$ be equal to 0 if the one of A and $F \backslash A$ that is true in ω receives alethic r_ω-rank equal to 0, and otherwise let it be equal to the positive alethic r_ω-rank of the one of A and $F \backslash A$ that is true in ω. However, I hasten to add that this is just to illustrate that the above is possible. And I also hasten to add that the singletons containing all the possible cases $\{\omega\}$ are not assigned one fixed actual rank for all factual antecedents A, but different actual ranks for different factual antecedents A. That is, we do not have one fixed model $(\Omega, rank)$ with a set of cases $\Omega = F \times R \times S$ plus an actual ranking function $rank$. Nor do we have a model $\left(\Omega, (rank_\omega)_{\omega\in\Omega}\right)$ with a set of cases Ω plus a family of actual ranking functions $rank_\omega$, one for each case $\omega \in \Omega$. Instead, we have a model $\left(\Omega, (rank_A)_{A\subseteq F}\right)$ with a set of possible cases Ω plus a family of actual ranking functions $rank_A$, one for each factual proposition $A \subseteq F$.

I will now show that, even if the royal rule is not satisfied, A-sensitivity to A is satisfied if the ideal doxastic agent is modally agnostic. Then I will show that A-adherence to A fails to be satisfied in a spectacular way even if the Stalnaker assumption obtains and the royal rule is satisfied. As to the former claim, what we need to show is that, given the assumption that all ϱ_ωs are modally agnostic, the following holds for all factual propositions $A \subseteq F$: every ω in which A is true and which is such that $rank_A(\{\omega\}) = rank_A(A \times R \times S)$ is also such that $\varrho_\omega(A \times R) = 0$.

Suppose we are in a case ω such that A is true in ω, and $\{\omega\}$ has A-minimal actual rank: $rank_A(\{\omega\}) = \min\left\{rank_A\left(\{\omega'\}\right) : [\![A]\!]_{\omega'} = true\right\}$. In such a case, by the above proposal, A has minimal alethic r_ω-rank: $r_\omega(A) = 0$. This is so, because, as shown above, there is at least one case ω in which A is true and which is such that A has alethic r_ω-rank 0. It follows that ϱ_ω does not disbelieve $A \times R$, provided she is modally agnostic. This is so, because in all ωs with $r_\omega(A) = 0$—and, indeed, any other ω!—we have:

$$\begin{aligned}\varrho_\omega(A \times R) &= \min\left\{\varrho_\omega\left((A \times R) \cap (r(A) = 0)\right), \varrho_\omega\left((A \times R) \cap (r(A) > 0)\right)\right\}\\ &= \min\left\{0, \varrho_\omega\left((A \times R) \cap (r(A) > 0)\right)\right\} = 0\end{aligned}$$

Now suppose the Stalnaker assumption obtains: in each possible case ω exactly one of A and $F \backslash A$ has positive alethic r_ω-rank. We are in a case ω where $r_\omega(A) = 0$ and hence $\varrho_\omega(F \backslash A) > 0$. Suppose further the royal rule is satisfied. Even then ϱ_ω *fails* to believe $A \times R$, provided she is modally agnostic. This is so, because in all ωs with $\varrho_\omega(F \backslash A) > 0$—and, indeed, any other ω!—we have:

$$\begin{aligned}\varrho_\omega\left((F \backslash A) \times R\right) &= \min\left\{\varrho_\omega\left((F \backslash A) \times R \cap (r(F \backslash A) = 0)\right),\right.\\ &\qquad \left.\varrho_\omega\left((F \backslash A) \times R \cap (r(F \backslash A) > 0)\right)\right\}\\ &= \min\left\{0, \varrho_\omega\left((F \backslash A) \times R \cap (r(F \backslash A) > 0)\right)\right\} = 0\end{aligned}$$

As an aside, the converse question, whether A would be true if it was believed, cannot be answered in the present setting, because only factual propositions $A \subseteq F$ have alethic ranks $r(A)$, whereas beliefs such as $\varrho(A \times C) = k$, or claims about alethic ranks such as $r(A) > n$ do not. So this peculiarity of Huber's (2014) set-up is not only making things complicated and inelegant. It also prevents the study of safety. In the appendix I will briefly sketch how one might study safety if one does not share Huber's (2014) worries about iterated modalities.

So far we have established that being modally agnostic is *a* sufficient means to attaining the cognitive end of believing in a sensitive way. The royal rule was not necessary for this result. However, this is a Pyrrhic victory, for the former assumption and the royal rule together imply that the ideal doxastic agent is also factually agnostic. By the royal rule for a non-empty factual proposition $A \subseteq F$:

$$\varrho((A \times R) \cap (r(A) = 0)) = 0 + \varrho(r(A) = 0) = 0 + 0 = 0.$$

This implies that the ideal doxastic agent who is modally agnostic and obeys the royal rule for every factual proposition also suspends judgment with respect to every contingent factual proposition. That is, for any factual proposition A such that $\emptyset \neq A \subseteq F$:

$$\varrho(A \times R) = \varrho((F \backslash A) \times R) = 0$$

The ideal doxastic agent would never believe anything that was false simply because she would never believe anything in the first place!

Modally agnostic agents achieve the cognitive goal described by A-sensitivity to A on the cheap. Therefore I will now drop this assumption and consider a different, but related cognitive end. I will show that obeying the royal rule for a factual proposition A is *a* sufficient means to attaining the cognitive end of believing A in an A-sensitive way for *all* ideal doxastic agents, not only those that are modally agnostic, *conditional on the truth about the modal status of A*. To this end consider the following conditions.

Conditional A-Sensitivity to A — For a *fixed* factual proposition $A \subseteq F$: If A were true, then the ideal doxastic agent would not disbelieve A conditional on the truth about the modal status of A, $r(A)^*$, in the sense that $\varrho\left(A \times R \mid r(A)^*\right) = 0$.

Conditional A-Adherence to A — For a *fixed* factual proposition $A \subseteq F$: If A were true, then the ideal doxastic agent would believe A conditional on the truth about the modal status of $F \backslash A$, $r(F \backslash A)^*$, in the sense that $\varrho\left((F \backslash A) \times R \mid r(F \backslash A)^*\right) > 0$.

First, a note on terminology and notation. $r_\omega(A)^*$ is any *purely modal*—i.e. if $(f, r) \in r_\omega(A)^*$ for some $f \in F$ and some $r \in R$, then $\left(f', r\right) \in r_\omega(A)^*$ for all $f' \in F$—proposition that is true in ω and accurately specifies the alethic rank of A as $r_\omega(A)$. Moreover, for a given case ω we quantify over *all* purely modal propositions $r_\omega(A)^*$

such that:

$$F \times \{r_\omega\} \subseteq r_\omega(A)^* \subseteq \{v = (f_v, r_v) \in F \times R : r_v(A) = r_\omega(A)\}.$$

Second, two notes on the condition $r(A)^*$ of the conditional belief $\varrho\left(A \times R \mid r(A)^*\right)$. The first note is that what is true about the modal status of A depends on which case ω we are considering. In the present framework the natural choice is the case ω the agent ϱ_ω is imagined to be in, as the actual ranks are not relativized to cases. This is different in the framework sketched in the appendix. The second note is that asking an agent to do whatever it takes to have sensitive and adherent beliefs in the sense of A-sensitivity to A and A-adherence to A is much like asking an agent to do whatever it takes to believe all and only true propositions: it is asking the agent to do something that is not within her reach. This is so despite my assumption that the agent is an ideal doxastic agent, which I take to mean that she does not suffer from any computational or other physical limitations, can always identify all logical and conceptual truths, gets to voluntarily decide what she believes, and never forgets any of her beliefs. Like ordinary agents, ideal doxastic agents do not foresee the future and are not omniscient.

Unlike such a request the royal rule is a rule that is within reach for ideal doxastic agents. The royal rule prescribes that an ideal doxastic agent hold various conditional beliefs, various beliefs conditional on various assumptions. The royal rule does not prescribe that the ideal doxastic agent hold a non-conditional belief in any contingent and purely factual proposition, or any contingent and purely modal proposition. This is somewhat similar to the situation for the different requirement that one's beliefs be logically consistent: this norm does not prescribe that the ideal doxastic agent hold a non-conditional belief in any contingent proposition, but only that she does not disbelieve A conditional on the assumption that she believes A.

A consequence of the fact that the royal rule is within an ideal doxastic agent's reach is that the ends it furthers are limited. Such is the nature of the means-end relationship: it goes both ways. There is no rule within a non-omniscient ideal doxastic agent's reach that is necessary and sufficient for holding all and only true beliefs in all possible worlds at all times. Something similar is true for A-sensitivity to A as we have seen. While it is within an ideal doxastic agent's reach to be modally agnostic, the royal rule then forces her to also be factually agnostic. Conditional A-sensitivity to A, on the other hand, is such a limited end that can be achieved without succumbing to agnosticism by obeying a norm that is within the ideal doxastic agent's reach. The reason is its conditional nature: the agent does not have to believe a true factual proposition, she merely has to refrain from disbelieving a true factual proposition conditional on what is true about this proposition's modal status in the case the agent is in.

Of course, the means-end relationship goes both ways, and what's within reach may not always be very desirable. That is, the conditional nature of conditional A-sensitivity to A may make this cognitive end appear to be too weak to be of interest. However, this is perhaps not the best way to think of it. Consider again consistency (now as an end rather than a means). Consistency alone won't sell a theory, but think about how hard it is to sell a theory that is not even consistent!

Similarly, beliefs that are merely sensitive conditional on the truth about the modal status of their contents may be hard to sell, but good luck selling beliefs that are not even sensitive conditional on the truth about the modal status of their contents! If your belief about the modal status of A is true, then having sensitive beliefs in the sense of conditional A-sensitivity to A guarantees that you would not disbelieve A if it were true. You get two items of potential value for the price of one: you would automatically, and for free, not disbelieve a true factual proposition once you managed to believe the truth about its modal status.[9]

However, if your beliefs are not sensitive in the sense of conditional A-sensitivity to A, then not even believing the truth about the modal status of A would automatically prevent you from disbelieving A if it were true. You would have to pay or work extra for this additional item of potential value.

Here is an example. Ann would not ever take a taxi without tipping the driver. Claire and Earl meet Ann and wonder if Ann will tip the driver given that she takes a taxi. Both Claire and Earl correctly believe that Ann would not take a taxi without tipping the driver. Claire's beliefs additionally satisfy the royal rule and so are conditionally sensitive: Claire believes that Ann will tip the driver given that Ann takes a taxi. Earl, on the other hand, does not believe that Ann will tip the driver given that she takes a taxi. His beliefs are not conditionally sensitive, and so do not satisfy the royal rule. The three friends meet and Ann tells them that she took a taxi, which in fact she did. As a consequence Claire and Earl come to correctly believe that Ann took a taxi.

So far so good. Now for Claire there is the following benefit of satisfying the royal rule and having conditionally sensitive beliefs. If Ann tipped the driver, then Claire would correctly refrain from believing otherwise, i.e. Claire would correctly refrain from believing that Ann did not tip the driver. Not so for Earl. If Ann tipped the driver, then Earl might incorrectly believe otherwise, i.e. he might incorrectly believe that Ann did not tip the driver.

So far for the example. Let us continue with the argument. To show that the royal rule is sufficient for conditional A-sensitivity to A, let us consider things in more detail. As before there are exactly two possibilities for a non-empty or consistent factual proposition A and its complement $F\backslash A$: A is true and $F\backslash A$ is false in ω, or it is the other way round. For each of these two possibilities there are many more possibilities: $r_\omega(A) = m$ and $r_\omega(F\backslash A) = n$, where at least one of m and n equals 0. And for each of these there are in turn also many more possibilities, or rather: entire arrays of possibilities $\overrightarrow{\varrho_\omega\left(A \times R \mid r_\omega(A)^*\right)} = \vec{k}$ and $\overrightarrow{\varrho_\omega\left((F\backslash A) \times R \mid r_\omega(F\backslash A)^*\right)} = \vec{l}$, where at least one of each component of the two vectors $\vec{k}$ and $\vec{l}$ equals 0. $\vec{k}$ and $\vec{l}$ are *vectors*, because there are *several* purely modal propositions $r_\omega(A)^*$ that truly specify the modal status of A in ω, and *for any single one of them* there is a conditional belief $\varrho_\omega\left(A \times R \mid r_\omega(A)^*\right)$. Similarly, there are *several* purely modal propositions $r_\omega(F\backslash A)^*$ that truly specify the modal status of $F\backslash A$ in ω, and *for any single one of them* there is a conditional belief $\varrho_\omega\left((F\backslash A) \times R \mid r_\omega(F\backslash A)^*\right)$.

[9] What to do in order to avoid believing a falsehood about the modal status of a factual proposition is explained in Huber (2015b).

All these possible cases can be represented as $\omega = (f_\omega, r_\omega, \varrho_\omega)$ or, stressing the relevant details, as:

$$\omega = \left([\![A]\!]_\omega, r_\omega(A), r_\omega(F \backslash A), \overrightarrow{\varrho_\omega\left(A \times R \mid r_\omega(A)^*\right)}, \overrightarrow{\varrho_\omega\left((F \backslash A) \times R \mid r_\omega(F \backslash A)^*\right)} \right)$$

In order to make sense of conditional A-sensitivity to A we now need to assign counterfactual distances to these ωs. As before, I propose that of two cases ω_1 and ω_2 that agree that the factual proposition A is true, whether or not the actual rank of $\{\omega_1\}$ qua case in which A is true is smaller than the actual rank of $\{\omega_2\}$ qua case in which A is true is determined by what their modal components r_{ω_1} and r_{ω_2} say about the modal status of A:

$$[\![A]\!]_{\omega_1} = [\![A]\!]_{\omega_2} = true \Rightarrow \left[rank_A(\{\omega_1\}) < rank_A(\{\omega_2\}) \Leftrightarrow r_{\omega_1}(A) < r_{\omega_2}(A)\right]$$

This is possible. For instance, we can let the actual rank of $\{\omega\}$ be equal to 0 if the one of A and $F \backslash A$ that is true in ω receives alethic r_ω-rank equal to 0, and otherwise let it be equal to the positive alethic r_ω-rank of the one of A and $F \backslash A$ that is true in ω.

As before, this is just to illustrate that the above is possible. Let me also stress again that the singletons containing the possible cases $\{\omega\}$ are not assigned one fixed actual rank for all factual antecedents A, but different actual ranks for different factual antecedents A.

I will now show that, for a fixed factual antecedent $A \subseteq F$, conditional A-sensitivity to A (and conditional A-adherence to A) is satisfied, if the royal rule holds for A. (This differs from the claim that conditional A-sensitivity to A holds for all A if the royal rule holds for all A, which is a point I will return to below.) What we need to show is that, given the royal rule for A, the following holds: every ω in which A is true and which is such that $rank_A(\{\omega\}) = rank_A(A \times R \times S)$ is also such that $\varrho_\omega\left(A \times R \mid r_\omega(A)^*\right) = 0$, for *any* purely modal proposition $r_\omega(A)^*$ that truly specifies the modal status of A in ω.

Suppose we are in a case ω such that A is true in ω, and $\{\omega\}$ has A-minimal actual rank, that is, actual rank equal to $\min\left\{rank_A\left(\{\omega'\}\right) : [\![A]\!]_{\omega'} = true\right\}$. In such a case, by the above proposal, A has minimal alethic r_ω-rank, that is, alethic r_ω-rank equal to 0. This is so, because, as shown above, there is at least one case ω in which A is true and which is such that A has alethic r_ω-rank 0. Therefore ϱ_ω does not disbelieve $A \times R$ conditional on any $r_\omega(A)^*$, provided she obeys the royal rule for A. This is so, because in all ωs with $r_\omega(A) = 0$ we have, for some admissible E that varies with $r_\omega(A)^*$:

$$\varrho_\omega\left(A \times R \mid r_\omega(A)^*\right) = \varrho_\omega(A \times R \mid (r(A) = 0) \cap E) = 0$$

Now suppose the Stalnaker-assumption holds: in each possible case ω exactly one of A and $F \backslash A$ has positive alethic r_ω-rank. We are in a case ω where $r_\omega(A) = 0$ and hence $\varrho_\omega(F \backslash A) = k$ for some $k > 0$. In such a case ϱ_ω believes $A \times R$ conditional on any $r_\omega(A)^*$, provided she obeys the royal rule for A. This is so, because in all ωs with $\varrho_{\omega}(F\backslash A)=k$ we have, for some admissible E that varies with $r_{\omega}(F\backslash A)^{*}$:

$$\varrho_{\omega}\left((F\backslash A)\times R\mid r_{\omega}(F\backslash A)^{*}\right)=\varrho_{\omega}((F\backslash A)\times R\mid(r(F\backslash A)=k)\cap E)=k>0$$

So far we have established that, for a fixed factual proposition $A\subseteq F$, obeying the royal rule for A is *a* sufficient means to attaining the cognitive end of believing A in an A-sensitive way *conditional on the truth about modal status of A* for *all* ideal doxastic agents (and to also do so in an adherent way, if, in contrast to the present paper, one makes the Stalnaker assumption). In order to not attribute too much strength to this claim it is important to remember that the notion of sensitivity is defined in terms of a counterfactual whose meaning varies with the antecedent A.

I will now reformulate the present result in an attempt to motivate a stronger condition that will be the official definition of tracking the facts. Then I will generalize this reformulation and show that, for a fixed factual proposition $A\subseteq F$, obeying the royal rule for A is not only *a* (or a sufficient) means, but *the only* (or a necessary) means to attaining the cognitive end of believing A in this generalized A-sensitive way conditional on the truth about the modal status of A. The royal rule for A, and it only, tracks the fact that A in this strong and conditional sense.

Consider the following reformulations of conditional A-sensitivity to A and conditional A-adherence to A:

Conditional *A-Sensitivity to A* — For a *fixed* factual proposition $A\subseteq F$: A case ω in which A is true, but in which the ideal doxastic agent ϱ_{ω} disbelieves A conditional on some truth about the modal status of A in ω in the sense that $\varrho_{\omega}\left(A\times R\mid r_{\omega}(A)^{*}\right)>0$, is bizarre, or counterfactually distant, i.e. $rank_{A}(\{\omega\})>rank_{A}(A\times R\times S)$.

Conditional *A-Adherence to A* — For a *fixed* factual proposition $A\subseteq F$: A case ω in which A is true, but in which the ideal doxastic agent ϱ_{ω} does not believe A conditional on some truth about modal status of $F\backslash A$ in ω in sense that $\varrho_{\omega}\left((F\backslash A)\times R\mid r_{\omega}(F\backslash A)^{*}\right)=0$, is bizarre, or counterfactually distant, i.e. $rank_{A}(\{\omega\})>rank_{A}(A\times R\times S)$.

These reformulations of conditional A-sensitivity to A and conditional A-adherence to A suggest the following strong and conditional version of sensitivity, which is my official definition of tracking the facts.

A-Tracking the Fact that A — For a *fixed* factual proposition $A\subseteq F$: Of two possible cases ω_1 and ω_2 in which A is true, the former is more bizarre, or counterfactually more distant, qua A-case than the latter if, and only if, in ω_1 the ideal doxastic agent disbelieves A conditional on the truth about its modal status in ω_1 to a high degree, whereas in ω_2 she (that is, her counterpart ϱ_{ω_2}) disbelieves A conditional

on the truth about its modal status in ω_2 to a low, and possibly no, degree, i.e.:
$\varrho_{\omega_1}\left(A \times R \mid r_{\omega_1}(A)^*\right) > \varrho_{\omega_2}\left(A \times R \mid r_{\omega_2}(A)^*\right) \Leftrightarrow rank_A(\{\omega_1\}) > rank_A(\{\omega_2\})$

Considerations analogous to the previous ones will establish that, for a fixed factual proposition $A \subseteq F$, obeying the royal rule for A is sufficient for attaining the cognitive end described by A-tracking the fact that A. The royal rule for A is not the only (not a necessary) means to attaining the cognitive end described by conditional A-sensitivity to A. This is only the case for A-tracking the fact that A, which logically implies conditional A-sensitivity to A.

Recall our example. Ann would not ever take a taxi without tipping the driver. Claire and Earl meet Ann and wonder if Ann will tip the driver given that she takes a taxi. Both Claire and Earl correctly believe that Ann would not take a taxi without tipping the driver. Claire's beliefs additionally satisfy the royal rule and so track the facts: Claire believes that Ann will tip the driver given that Ann takes a taxi. Earl, on the other hand, does not believe that Ann will tip the driver given that she takes a taxi. His beliefs do not track the facts, and so do not satisfy the royal rule. The three friends meet and Ann tells them that she took a taxi, which in fact she did. As a consequence Claire and Earl come to correctly believe that Ann took a taxi.

As noted earlier, for Claire there is the following benefit of satisfying the royal rule and having beliefs that track the facts. If Ann tips the driver, but Claire incorrectly believes otherwise, no matter how weakly, then this is a bizarre situation: a situation that would not obtain if some situation obtained. Not so for Earl. If Ann tips the driver, but Earl incorrectly believes otherwise, no matter how weakly, then this is a situation that is not bizarre: it is not a situation that would not obtain if some situation obtained. Furthermore, if Ann tips the driver, but Claire incorrectly believes otherwise, and she does so firmly, then this is an outlandish situation: a situation that would not obtain even if a situation obtained that would not obtain if some situation obtained.

What are such situations like that are not merely bizarre, but outlandish—situations that would not obtain, even if a situation obtained that would not obtain if some situation obtained? They are like the following ones. One should not steal. However, given that one steals, one should steal from the rich and not the poor. Ann is a good person. She would not steal. Moreover, if she stole, should would steal from the rich and not the poor. A situation in which Ann steals is bizarre. A situation in which Ann steals from the poor and not the rich is not merely bizarre, but outlandish.

5.1 Sufficiency of the royal rule for tracking the facts

There are exactly two possibilities for a non-empty or consistent factual proposition A and its complement $F \backslash A$: A is true and $F \backslash A$ is false in ω, or it is the other way round. For each of these two possibilities there are many more possibilities: $r_\omega(A) = m$ and $r_\omega(F \backslash A) = n$, where at least one of m and n equals 0. And for each of these there are in turn also many more arrays of possibilities: $\overrightarrow{\varrho_\omega\left(A \times R \mid r_\omega(A)^*\right)} = \overrightarrow{k}$ and $\overrightarrow{\varrho_\omega\left((F \backslash A) \times R \mid r_\omega(F \backslash A)^*\right)} = \overrightarrow{l}$, where at least one of each component of the two vectors $\overrightarrow{k}$ and $\overrightarrow{l}$ equals 0.

All these possible cases can be represented as $\omega = (f_\omega, r_\omega, \varrho_\omega)$ or, stressing the relevant details, as:

$$\omega = \left(\llbracket A \rrbracket_\omega, r_\omega(A), r_\omega(F \backslash A), \overrightarrow{\varrho_\omega\left(A \times R \mid r_\omega(A)^*\right)}, \overrightarrow{\varrho_\omega\left((F \backslash A) \times R \mid r_\omega(F \backslash A)^*\right)}\right)$$

We need to assign counterfactual distances to these ωs. Once more I propose that of two cases ω_1 and ω_2 that agree that the factual proposition A is true, whether or not the actual rank of $\{\omega_1\}$ qua A-case is smaller than the actual rank of $\{\omega_2\}$ qua A-case is determined by what their modal components r_{ω_1} and r_{ω_2} say about the modal status of A:

$$\llbracket A \rrbracket_{\omega_1} = \llbracket A \rrbracket_{\omega_2} = true \Rightarrow \left[rank_A(\{\omega_1\}) < rank_A(\{\omega_2\}) \Leftrightarrow r_{\omega_1}(A) < r_{\omega_2}(A)\right]$$

This is possible. For instance, we can let the actual rank of $\{\omega\}$ be equal to 0 if the one of A and $F \backslash A$ that is true in ω receives alethic r_ω-rank equal to 0, and otherwise let it be equal to the positive alethic r_ω-rank of the one of A and $F \backslash A$ that is true in ω. Again, I hasten to add that this is just to illustrate that the above is possible. And I repeat that the singletons containing the possible cases $\{\omega\}$ are not assigned one fixed actual rank for all factual antecedents A, but different actual ranks for different factual antecedents A. This is perhaps clearest by recalling that $rank_A$ specifies the bizarreness of various cases qua cases in which A is true.

Consider ω_1 and ω_2 and suppose A is true in both ω_1 and ω_2, and both ϱ_{ω_1} and ϱ_{ω_2} satisfy the royal rule for A. Then we have, for suitable E_1 and E_2 that depend on $r_{\omega_1}(A)^*$ and $r_{\omega_2}(A)^*$, respectively:

$$\begin{array}{rcl} \varrho_{\omega_1}\left(A \times R \mid r_{\omega_1}(A)^*\right) & > & \varrho_{\omega_2}\left(A \times R \mid r_{\omega_2}(A)^*\right) \\ & \Leftrightarrow & \\ \varrho_{\omega_1}\left(A \times R \mid \left(r(A) = r_{\omega_1}(A)\right) \cap E_1\right) & > & \varrho_{\omega_2}\left(A \times R \mid \left(r(A) = r_{\omega_2}(A)\right) \cap E_2\right) \\ & \Leftrightarrow & \\ r_{\omega_1}(A) & > & r_{\omega_2}(A) \\ & \Leftrightarrow & \\ rank_A(\{\omega_1\}) & > & rank_A(\{\omega_2\}) \end{array}$$

5.2 Necessity of the royal rule for tracking the facts

Suppose there is a factual proposition $A \subseteq F$ and some claim abouts its alethic rank $r(A) = m$ and some admissible proposition E, i.e. a purely modal proposition E that is consistent with $r(A) = m$, such that the ideal doxastic agent's ranking function violates the royal rule for A: $\varrho(A \times R \mid (r(A) = m) \cap E) = s \neq m$.

If $s > m$, then there are ω_1, where the royal rule for A is violated, and ω_2, where the royal rule for A is not violated, such that: $[\![A]\!]_{\omega_1} = [\![A]\!]_{\omega_2} = true$ and $r_{\omega_1}(A) = r_{\omega_2}(A) = m$ and $F \times \{r_{\omega_1}, r_{\omega_2}\} \subseteq E$ so that E is not only purely modal, but also true in both ω_1 and ω_2. For the following purely modal truth in both ω_1 and ω_2 about the modal status of A: $(r(A) = m) \cap E$, which thus is both a truth about the modal status of A in ω_1, $r_{\omega_1}(A)^*$, as well as a truth about the modal status of A in ω_2, $r_{\omega_2}(A)^*$, we obtain a violation of A-tracking the fact that A:

$$\begin{aligned}
\varrho_{\omega_1}\left(A \times R \mid r_{\omega_1}(A)^*\right) &= \varrho_{\omega_1}(A \times R \mid (r(A) = m) \cap E) \\
&= s \\
&> m \\
&= \varrho_{\omega_2}(A \times R \mid (r(A) = m) \cap E) \\
&= \varrho_{\omega_2}\left(A \times R \mid r_{\omega_2}(A)^*\right)
\end{aligned}$$

This is so, even though $rank_A(\{\omega_1\}) = rank_A(\{\omega_2\}) = m$. If $s < m$ it is the other way round.

This completes my argument for the thesis that an ideal doxastic agent ought to obey the royal rule if she aims at attaining the cognitive end of tracking the facts: for a fixed factual proposition $A \subseteq F$, the royal rule for A, and it only, A-tracks the fact that A. The objection that the ideal doxastic agent may not have this cognitive end cuts no ice, but mistakes a hypothetical imperative for a categorical one. Needless to say, if the ideal doxastic agent does not have this cognitive end, then there is nothing wrong with her. To each their own.

6 Conclusion

In concluding let me state once more, but in slightly different terms, what I have shown, and what I have not shown. I have not shown that an ideal doxastic agent's subjective ranking function ϱ obeys the royal rule for all factual propositions $A \subseteq F$ if and only if ϱ tracks tracks the truth of all factual propositions $A \subseteq F$,

for all factual propositions $A \subseteq F$: ϱ satisfies the royal rule for A
if, and only if,
for all factual propositions $A \subseteq F$: ϱ tracks the truth of A.

What I have shown is that, for all factual propositions $A \subseteq F$, an ideal doxastic agent's subjective ranking function ϱ obeys the royal rule for A if, and only if, ϱ A-tracks the truth of A,

for all factual propositions $A \subseteq F$:
ϱ satisfies the royal rule for A if, and only if, ϱ A-tracks the truth of A.

Tracking is defined in terms of a counterfactual. The meaning of a counterfactual depends on a ranking function that specifies the counterfactual distance of various cases. On the present approach this ranking function varies with the antecedent of a counterfactual. To stress this dependence on the antecedent A I have used the phrase 'A-tracks the fact that A' rather than 'tracks the fact that A.'

For a notion of tracking on which there is no difference between A-tracking and B-tracking the second claim above logically implies the first claim above. However, this is not the case for the claim established in this paper. While the following are not cases that the present paper has considered, there is a difference between A-tracking the fact that C and B-tracking that very same fact C.[10] Ultimately this feature is due to Huber's (2014) framework which distinguishes between the factual component and the alethically modal component of a possible world. The appendix sketches a framework where this distinction is not made and where a notion of tracking can be defined that does not vary with the antecedent of the counterfactual figuring in its definition.

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Appendix

Someone interested in studying safety, and not bothered by Huber's (2014) thought that it is important to distinguish between the factual components f_w and the modal components r_w of a possible world w, will find the following models more elegant: $\left(W, (R_w)_{w\in W}, (S_w)_{w\in W}\right)$, where W is a non-empty set of possible worlds, $(R_w)_{w\in W}$ is a family of *arbitrary* functions from $\wp(W)$ into $\mathbb{N}\cup\{\infty\}$, and $(S_w)_{w\in W}$ is a family of regular subjective ranking functions from $\wp(W)$ into $\mathbb{N}\cup\{\infty\}$.

In this set-up the royal rule says: for all $v \in W$ and all $A \subseteq W$, and all "v, A-admissible" propositions $E \subseteq W$,

$$S_v\left(A \mid \{w \in W : R_w(A) = n\} \cap E\right) = n.$$

With the right assumptions about admissibility it follows from these assumptions that R_w is a ranking function for each $w \in W$. In particular, the condition from sections 3 and 4 that r_w be a ranking function is not an assumption of the present paper, but rather a *consequence* of the royal rule that I have taken for granted to simplify the presentation of this paper (for a derivation see Huber 2014).

In this set-up, instead of considering cases ω one considers possible worlds $w = (\llbracket\rrbracket_w, R_w, S_w)$. Sensitivity says: for all $v \in W$ and all $A \subseteq W$, all A-worlds w that are R_v-minimal are such that $S_w(A) = 0$. Adherence says: for all $v \in W$ and all $A \subseteq W$, all A-worlds w that are R_v-minimal are such that $S_w(W\setminus A) > 0$. Safety says: for all $v \in W$ and all $A \subseteq W$, A is true in all R_v-minimal worlds w in which $S_w(W\setminus A) > 0$.

One conditional version of sensitivity says: for all $v \in W$ and all $A \subseteq W$, all A-worlds w that are R_v-minimal are such that

$$S_w\left(A \mid \{x \in W : R_x(A) = R_w(A)\} \cap E\right) = 0$$

[10] Obviously the second claim that has been established logically implies the claim that results from the first claim by substituting 'ϱ A-tracks the truth of A' for 'ϱ tracks the truth of A.'

for all "w, A-admissible" $E \subseteq W$. One conditional version of adherence says: for all $v \in W$ and all $A \subseteq W$, all A-worlds w that are R_v-minimal are such that

$$S_w (W \backslash A \mid \{x \in W : R_x (W \backslash A) = R_w (W \backslash A)\} \cap E) > 0$$

for all "w, $(W \backslash A)$-admissible" $E \subseteq W$. One conditional version of safety says: for all $v \in W$ and all $A \subseteq W$, A is true in all R_v-minimal worlds w in which

$$S_w (W \backslash A \mid \{x \in W : R_x (W \backslash A) = R_w (W \backslash A)\} \cap E) > 0$$

for some "w, $(W \backslash A)$-admissible" $E \subseteq W$.

These conditional versions consider the truth about the modal status of A in the word w the agent is in. Quite *different* conditional versions consider the truth about the modal status of A in the world v where the counterfactual conditional is evaluated. The conditions studied in this paper considered if the agent believes A in w conditional on what is true about the modal status of A *in the world w she is in*. The different conditions consider if she believes A in w conditional on what is true about the modal status of A *elsewhere, in a different possible world v*.

One way to think of the former conditions is in terms of what is true from the first person perspective of the agent, and to think of the latter conditions in terms of what is true from the third person perspective of some tracking-ascriber. The latter conditional versions of sensitivity and adherence and safety say: for all $v \in W$ and all $A \subseteq W$, all A-worlds w that are R_v-minimal are such that $S_w (A \mid \{x \in W : R_x (A) = R_v (A)\} \cap E) = 0$ for all "v, A-admissible" $E \subseteq W$. And: for all $v \in W$ and all $A \subseteq W$, all A-worlds w that are R_v-minimal are such that $S_w (W \backslash A \mid \{x \in W : R_x (W \backslash A) = R_v (W \backslash A)\} \cap E) > 0$ for all "v, $(W \backslash A)$-admissible" $E \subseteq W$. As well as: for all $v \in W$ and all $A \subseteq W$, A is true in all R_v-minimal worlds w in which $S_w (W \backslash A \mid \{x \in W : R_x (W \backslash A) = R_v (W \backslash A)\} \cap E) > 0$ for some "v, $(W \backslash A)$-admissible" $E \subseteq W$.

Bayesians may want to interpret R_w as the objective chance distribution of world w, ch_w, and S_w as the ideal doxastic agent's subjective credence function in w, cr_w (as well as change the domain and range to some suitable algebra as well as the unit interval, respectively). On this probabilistic interpretation the royal rule becomes Lewis' (1980) principal principle, and the probabilistically re-interpreted set-up allows one to study which ends the principal principle furthers (Pettigrew 2013 is an excellent study of this kind). In addition one can study which norms further the following cognitive ends that also consider what is true about the chance of A from the third person perspective mentioned above:

- The objective chance is high that one's subjective credence in A conditional on the truth about the objective chance of A is high given that A is true. For all $A \subseteq W$ and all $v \in W$ and all "v, A-admissible" $E \subseteq W$,
  $ch_v (cr (A \mid \{x \in W : ch_x (A) = ch_v (A)\} \cap E) = high \mid A) = high$.
- The objective chance equals c that A is true given that one's subjective credence in A conditional on the truth about the objective chance of A equals c. For all $A \subseteq W$ and all $v \in W$ and all " v, A-admissible" $E \subseteq W$,
  $ch_v (A \mid cr (A \mid \{x \in W : ch_x (A) = ch_v (A)\} \cap E) = c) = c$.

Here $cr(A \mid C) = high$ is the proposition $\{x \in W : cr_x(A \mid C) = high\}$.

Of course, in this more elegant and general set-up, it is not possible anymore to say that admissible information is purely modal information, as we have done in this paper. The reason is that, in this more elegant and general set-up, the very distinction between factual information and modal information cannot be drawn without further assumptions. Admissibility now is relative to a proposition and a world (and, of course, a credence function), and this is only the tip of the iceberg that is known as the "big bad bug" (Rachael 2009) and that is the reason for Huber's (2014) complicated set-up in the first place.[11]

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[11] There are, of course, also the unconditional versions of probabilistic sensitivity and safety, respectively. The objective chance is high that one's subjective credence in A is high given that A is true. For all $A \subseteq W$ and all $v \in W$, $ch_v(\{x \in W : cr_x(A) = high\} \mid A) = high$. The objective chance equals c that A is true given that one's subjective credence in A equals c. For all $A \subseteq W$ and all $v \in W$, $ch_v(A \mid \{x \in W : cr_x(A) = c\}) = c$.

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